Network-Structured Particle Swarm Optimizer with Various Topology and Its Behaviors

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Abstract. This study proposes Network-Structured Particle Swarm Optimizer (NS-PSO) with various neighborhood topology. The proposed PSO has the various network topology as rectangular, hexagonal, cylinder and toroidal. We apply NS-PSO with various topology to optimization problems. We investigate their behaviors and evaluate what kind of topology would be the most appropriate for each function.

Keywords: Particle Swarm Optimization (PSO), network structure, Self-Organizing Map (SOM).

1 Introduction

Particle Swarm Optimization (PSO) [1] is an evolutionary algorithm to simulate the movement of flocks of birds. Due to the simple concept, easy implementation and quick convergence, PSO has attracted much attention and is used to wide applications in different fields in recent years. In PSO algorithm, there are no special relationships between particles. Each particle position is updated according to its personal best position and the best particle position among the all particles, and their weights are determined at random in every generation.

On the other hand, the Self-Organizing Map (SOM) [2] is an unsupervised learning and is a simplified model of the self-organizing process of the brain. The map consists of neurons located on a hexagonal or rectangular grid. The neurons self-organize statistical features of the input data according to the neighborhood relationship of the map structure.

Various topological neighborhoods for PSO have been considered by researches [3]–[7]. Each particle shares its best position among neighboring particles on the network. However, the information of each particle is not updated according to the neighborhood distance on the network.

In our past study, we have applied the concept of SOM to PSO and have proposed a new PSO algorithm with topological neighborhoods; Network-Structured Particle Swarm Optimizer considering neighborhood relationships (NS-PSO) [8]. All particles of NS-PSO are connected to adjacent particles by a neighborhood relation, which dictates the topology of the 2-dimensional network. The connected particles, namely neighboring particles on the network, share the information of

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their own best position. In every generation, we find a winner particle, whose function value is the best among all particles, as SOM algorithm, and each particle is updated depending on the neighborhood distance between it and the winner on the network. However, the relevance between the efficiently of optimization and the shape of network topology of NS-PSO was not completely clear.

In this study, we propose NS-PSO with various neighborhood topology. We apply NS-PSO to the various network topology as rectangular, hexagonal, cylinder and toroidal. NS-PSO with various topology are applied to eight test functions which are unimodal and multimodal. We investigate their behaviors and evaluate what kind of topology would be the most appropriate for each function. From results, we find that the circular-topology is effective for the simple unimodal functions, because this topology easily transmits the information of each best position to the whole particles. We also confirm that the hexagonal-topology is appropriate for the complex multimodal functions, because this topology contains various kinds of particles and this effect averts the premature convergence.

2 Network-Structured Particle Swarm Optimizer Considering Neighborhood Relationships (NS-PSO)

In the algorithm of the standard PSO, multiple solutions called "particles" coexist. At each time step, the particle flies toward its own past best position and the best position among all particles. Each particle has two informations; position and velocity. The position vector of each particle i and its velocity vector are represented by $\boldsymbol{X}_i = (x_{i1}, \cdots, x_{id}, \cdots, x_{iD})$ and $\boldsymbol{V}_i = (v_{i1}, \cdots, v_{id}, \cdots, v_{iD})$, respectively, where $(d = 1, 2, \cdots, D)$, $(i = 1, 2, \cdots, M)$ and $x_{id} \in [x_{\min}, x_{\max}]$.

The algorithm of NS-PSO is based on both two structures; the standard PSO and SOM. NS-PSO has following three key features.

1. All particles are connected to adjacent particles by a neighborhood relation which dictates the topology of the network. In this study, we use various topology networks shown in Fig. 1 and investigate their behaviors. The rectangular-topology and the hexagonal-topology as Figs. 1(a)–(b) are the sheet shapes, and the cylinder-topology and toroidal-topology as Figs. 1(c)–(d) are circular map.

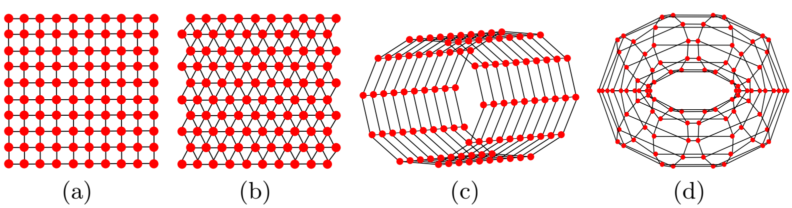


Fig. 1. Different map shapes with 10×10 particles used in this study. (a) Rectangular-topology. (b) Hexagonal-topology. (c) Cylinder-topology. (d) Toroidal-topology.

2. The particles share the local best position between the neighborhood particles directly connected.
3. In every generation, we find a winner particle with best function value among all particle as SOM learning.

By these features, each particle of NS-PSO is updated depending on its own best position, the position of the winner and the neighborhood distance between it and the winner on the network.

(NS-PSO1) (Initialization) Let a generation step $t = 0$. Randomly initialize the particle position $\boldsymbol{X}_i$, initialize its velocity $\boldsymbol{V}_i$ for each particle i to zero, and initialize $\boldsymbol{P}_i = (p_{i1}, p_{i2}, \cdots, p_{iD})$ with a copy of $\boldsymbol{X}_i$. Evaluate the objective function $f(\boldsymbol{X}_i)$ for each particle i and find $\boldsymbol{P}_g$ with the best function value among all the particles. Define g as the winner c. Find $\boldsymbol{L}_i = (l_{i1}, l_{i2}, \cdots, l_{iD})$ with the best function value among the directly connected particles, namely own neighbors.

(NS-PSO2) Evaluate the fitness $f(\boldsymbol{X}_i)$ and find a winner particle c with the best fitness among the all particles at current time t;

$$c = \arg\min_i \{f(\boldsymbol{X}_i(t))\}. \tag{1}$$

For each particle i, if $f(\boldsymbol{X}_i) < f(\boldsymbol{P}_i)$, the personal best position (called *pbest*) $\boldsymbol{P}_i = \boldsymbol{X}_i$. Let $\boldsymbol{P}_g$ represents the best position with the best fitness among all particles so far (called *gbest*). If $f(\boldsymbol{X}_c) < f(\boldsymbol{P}_g)$, update *gbest* $\boldsymbol{P}_g = \boldsymbol{X}_c$, where $\boldsymbol{X}_c = (x_{c1}, x_{c2}, \cdots, x_{cD})$ is the position of the winner c.

(NS-PSO3) Find each local best position (called *lbest*) $\boldsymbol{L}_i$ among the particle i and its neighborhoods, which are directly connected with i on the network, so far. For each particle i, update *lbest* $\boldsymbol{L}_i$, if needed.

(NS-PSO4) Update $\boldsymbol{V}_i$ and $\boldsymbol{X}_i$ of each particle i depending on its *lbest*, position of the winner $\boldsymbol{X}_c$ and the distance on the network between i and the winner c, according to

$$\begin{aligned} v_{id}(t+1) &= w v_{id}(t) + c_1 \text{rand}(\cdot)\left(l_{id} - x_{id}(t)\right) + c_2 h_{c,i}\left(x_{cd} - x_{id}(t)\right), \\ x_{id}(t+1) &= \ x_{id}(t) + v_{id}(t+1), \end{aligned} \tag{2}$$

where w is the inertia weight determining how much of the previous velocity of the particle is preserved. c_1 and c_2 are two positive acceleration coefficients, generally $c_1 = c_2$, $\text{rand}(\cdot)$ is an uniform random number sample from $U(0,1)$. $h_{c,i}$ is the fixed neighborhood function defined by

$$h_{c,i} = \exp\left(-\frac{\|\boldsymbol{r}_i - \boldsymbol{r}_c\|^2}{2\sigma^2}\right), \tag{3}$$

where $\|\boldsymbol{r}_i - \boldsymbol{r}_c\|$ is the distance between network nodes c and i on the network, and the fixed parameter σ corresponds to the width of the neighborhood function.

Therefore, large σ strengthens particles' spreading force to the whole space, and small σ strengthens their convergent force toward the winner.

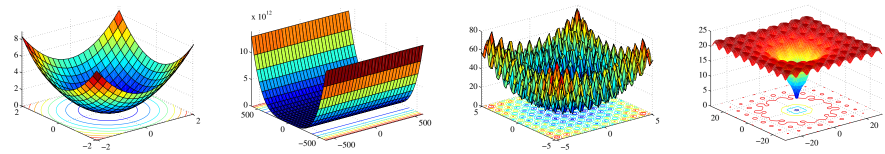
(NS-PSO5) Let $t = t + 1$ and go back to (NS-PSO2).

3 Experimental Results

In order to evaluate the performance of NS-PSO with various topology, we use eight benchmark optimization problems summarized in Table 1. f_1, f_2, f_3 and f_4 are unimodal functions, and f_5, f_6, f_7 and f_8 are multimodal functions with numerous local minima. All the functions have D variables, and the symmetric landscape maps of Sphere, Rosenbrock, Rastrigin and Ackley functions with two variables are shown in Fig. 2. Table 2 lists the dimensionality D, the optimum solution x^*, the optimum function value $f(x^*)$ and the initialization ranges. In order to investigate the behaviors in various initialization spaces, we use the symmetric and the asymmetric initialization spaces. The population size M is set to 36 in PSO, and the network size is 6×6 in NS-PSO with each topology. For PSO and NS-PSO, the parameters are set as $w = 0.7$ and $c_1 = c_2 = 1.6$. The neighborhood radius σ of all NS-PSOs are 1.5. We carry out the simulations repeated 30 times for all the optimization functions with 3000 generations.

Table 1. Eight Test Functions

Function name	Test Function
Sphere function;	$f_1(x) = \sum_{d=1}^{D-1} x_d^2$
Rosenbrock's function;	$f_2(x) = \sum_{d=1}^{D-1} \left(100\left(x_d^2 - x_{d+1}\right)^2 + (1 - x_d)^2\right)$
3rd De Jong's function;	$f_3(x) = \sum_{d=1}^{D} \lvert x_d \rvert$
4th De Jong's function;	$f_4(x) = \sum_{d=1}^{D} d x_d^4$
Rastrigin's function;	$f_5(x) = \sum_{d=1}^{D} \left(x_d^2 - 10\cos(2\pi x_d) + 10\right)$
Ackley's function;	$f_6(x) = \sum_{d=1}^{D-1} \left(20 + e - 20e^{-0.2\sqrt{0.5(x_d^2 + x_{d+1}^2)}} - e^{0.5(\cos(2\pi x_d) + \cos(2\pi x_{d+1}))}\right)$
Stretched V sine wave;	$f_7(x) = \sum_{d=1}^{D-1} (x_d^2 + x_{d+1}^2)^{0.25} \left(1 + \sin^2(50(x_d^2 + x_{d+1}^2)^{0.1})\right)$
Griewank's function;	$f_8(x) = \sum_{d=1}^{D} \frac{x_d^2}{4000} - \prod_{d=1}^{D} \cos\left(\frac{x_d}{\sqrt{d}}\right) + 1$



(a) (b) (c) (d)

Fig. 2. Symmetric landscape of four test functions with two variables. First and second variables are on the x-axis and y-axis, respectively, and z-axis shows its function value. (a) Sphere. (b) Rosenbrock. (c) Rastrigin. (d) Ackley.

Table 2. Parameters for test functions

f	D	x^*	$f(x^*)$	Initialization Space	
				Symmetric	Asymmetric
f_1	50	$[0, 0, \ldots, 0]$	0	$[-5.12, 5.12]^D$	$[-2.56, 5.12]^D$
f_2	50	$[1, 1, \ldots, 1]$	0	$[-2.048, 2.048]^D$	$[-1.024, 2.048]^D$
f_3	50	$[0, 0, \ldots, 0]$	0	$[-2.048, 2.048]^D$	$[-1.024, 2.048]^D$
f_4	50	$[0, 0, \ldots, 0]$	0	$[-1.28, 1.28]^D$	$[-0.64, 1.28]^D$
f_5	50	$[0, 0, \ldots, 0]$	0	$[-5.12, 5.12]^D$	$[-2.56, 5.12]^D$
f_6	50	$[0, 0, \ldots, 0]$	0	$[-10, 10]^D$	$[-5, 10]^D$
f_7	50	$[0, 0, \ldots, 0]$	0	$[-30, 30]^D$	$[-15, 30]^D$
f_8	50	$[0, 0, \ldots, 0]$	0	$[-600, 600]^D$	$[-300, 600]^D$

3.1 Symmetric and Asymmetric Functions

The performances with the minimum and mean function values over 30 independent runs on eight functions with the symmetric initialization are listed in Table 3. The best results of the mean values among all the algorithms are shown in bold. All NS-PSOs with various topology evidently surpasses the standard PSO on all the eight functions. In fact, the standard PSO has not obtained better results than any other algorithms which consider the network-structure. From these results, we can say that PSO, which has the specific network-structure, is more effective than the standard PSO, which has no neighborhood relationship, for the symmetric functions.

Table 4 shows the best result among the five algorithms and the difference between the best result and the result of each algorithm. NS-PSO with rectangular-topology, with hexagonal-topology, with cylinder-topology and with toroidal-topology achieve the best values 0, 3, 2 and 3 times, respectively. For the unimodal functions as f_1, f_2, f_3 and f_4, NS-PSO with toroidal-topology has obtained the best results most frequently, and the cylinder-topology delivers a very small difference from the best results. However, for the multimodal functions as f_5, f_6, f_7 and f_8, the differences between the results of toroidal-topology and the best results are bigger than other three NS-PSOs although it is the best topology for the unimodal functions. Meanwhile, NS-PSO with hexagonal-topology

Table 3. Comparison results of PSO and NS-PSO with symmetric initialization on 8 test functions with $D = 50$

f		PSO	NS-PSO			
			Rectangular	Hexagon	Cylinder	Toroidal
f_1	Mean	2.29e-20	8.22e-25	1.50e-23	1.62e-25	**1.58e-25**
	Minimum	4.09e-27	1.51e-29	1.29e-22	9.59e-32	1.78e-31
f_2	Mean	55.24	43.61	42.56	**38.80**	40.82
	Minimum	36.74	38.48	35.56	31.04	30.33
f_3	Mean	7.49e-06	1.23e-07	**7.37e-09**	3.93e-08	4.50e-08
	Minimum	9.41e-11	3.15e-12	5.81e-13	1.85e-12	3.19e-11
f_4	Mean	1.58e-35	1.51e-41	1.32e-41	2.90e-42	**3.53e-44**
	Minimum	9.86e-42	7.96e-47	2.84e-46	3.90e-49	1.32e-50
f_5	Mean	148.31	92.80	104.44	**88.32**	115.45
	Minimum	94.52	52.73	60.69	45.77	29.85
f_6	Mean	249.67	159.62	**157.28**	193.50	205.75
	Minimum	97.84	67.60	41.13	64.90	66.46
f_7	Mean	65.62	41.35	**33.46**	41.06	43.04
	Minimum	39.36	21.95	17.68	18.90	21.78
f_8	Mean	0.2440	0.0853	0.0448	0.0924	**0.0350**
	Minimum	0	0	1.11e-16	1.11e-16	0

Table 4. Difference from the best result with symmetric initialization

f	Best Mean Result	Difference from the best mean result				
		PSO	NS-PSO			
			Rectangular	Hexagon	Cylinder	Toroidal
f_1	1.58e-25	2.29e-20	6.64e-25	1.49e-23	4.72e-27	0
f_2	38.80	16.44	4.82	3.77	0	2.02
f_3	7.37e-09	7.48e-06	1.15e-07	0	3.19e-08	3.76e-08
f_4	3.53e-44	1.58e-35	1.50e-41	1.32e-41	1.32e-41	0
f_5	88.319	60.00	4.48	16.12	0	27.13
f_6	157.28	92.39	2.34	0	36.22	48.47
f_7	33.4615	32.16	7.89	0	7.60	9.58
f_8	0.0350	0.2090	0.0503	0.0098	0.0574	0

obtains the best results on f_6 and f_7, and it can obtain stable good results, which are small differences from the best results, for other two multimodal functions. NS-PSO with rectangular-topology achieves the stable good results for both the unimodal and multimodal functions even if it can not obtain the best results among NS-PSOs for any benchmarks.

The performances over 30 independent runs on asymmetric functions are listed in Table 5. Since the standard PSO can not obtain the best results among all five PSOs for any benchmarks, PSO with some networks is more suitable for the optimization problems than the standard PSO.

Table 5. Comparison results of PSO and NS-PSO with asymmetric initialization on 8 test functions with $D = 50$

f		PSO	NS-PSO			
			Rectangular	Hexagon	Cylinder	Toroidal
f_1	Mean	2.31e-21	2.03e-24	1.13e-23	4.95e-22	**8.15e-26**
	Minimum	3.58e-26	2.96e-29	8.27e-28	3.99e-30	8.90e-31
f_2	Mean	55.96	64.80	50.02	65.96	**40.23**
	Minimum	6.22	15.47	0.1998	13.19	0.2033
f_3	Mean	2.06e-05	2.56e-08	**8.32e-09**	7.60e-08	1.02e-07
	Minimum	1.65e-10	5.86e-12	3.29e-12	2.38e-11	1.15e-11
f_4	Mean	4.82e-36	4.72e-43	3.60e-39	3.44e-43	**4.02e-44**
	Minimum	8.51e-41	2.74e-47	1.79e-46	1.41e-48	8.03e-51
f_5	Mean	150.70	96.54	89.65	**87.16**	153.61
	Minimum	104.47	57.71	49.75	53.73	34.63
f_6	Mean	190.24	177.03	**142.66**	207.75	217.90
	Minimum	69.48	69.13	37.87	81.76	35.02
f_7	Mean	61.90	42.19	**34.93**	37.18	41.14
	Minimum	37.86	21.84	19.00	21.87	18.33
f_8	Mean	0.0521	**0.0240**	0.1199	0.0249	0.1576
	Minimum	0	0	0	0	1.11e-16

Table 6. Difference from the best result with asymmetric initialization

f	Best Mean Result	Difference from the best mean result				
		PSO	NS-PSO			
			Rectangular	Hexagon	Cylinder	Toroidal
f_1	8.15e-26	2.31e-21	1.95e-24	1.13e-23	4.95e-22	0
f_2	40.23	15.73	24.57	9.79	25.73	0
f_3	8.32e-09	2.06e-05	1.73e-08	0	6.77e-08	9.38e-08
f_4	4.02e-44	4.82e-36	4.32e-43	3.60e-39	3.04e-43	0
f_5	87.16	63.54	9.39	2.49	0	66.45
f_6	142.66	47.58	34.37	0	65.09	75.24
f_7	34.93	26.96	7.26	0	2.25	6.21
f_8	0.0240	0.0281	0	0.0959	9.13e-04	0.1336

Table 6 shows the the difference between the best result and the result of each algorithm. For the unimodal functions, NS-PSO with toroidal-topology can obtain the best results on f_1, f_2 and f_4, and also on f_3, it is a very small difference from the best results. Therefore, we can say that toroidal-topology is the most effective for the asymmetric unimodal functions as same as the symmetric unimodal functions. However, for the multimodal functions, NS-PSO with toroidal-topology obtain the worst results 3 times among five algorithms including the standard PSO. On the other hand, NS-PSO with hexagonal-topology obtains the best results 2 times, in particular, it evidently surpasses other four algorithms on f_6.

From these results, on both symmetric and asymmetric spaces, the circular-shaped NS-PSO as toroidal-topology is more suitable for the unimodal functions, and the sheet-shaped NS-PSO as hexagonal-topology is more effective for the multimodal functions. In particular, we found that the toroidal-topology is not suitable on the asymmetric multimodal functions.

3.2 Behaviors of NS-PSO with Various Topology

The convergence rate of NS-PSO is almost same or slower than the standard PSO. In the standard PSO, the particles move toward *gbest* or toward *pbest*, however, the direction, which more particles move toward, is decided at random on every generation. On the other hand, the neighborhood gaussian function is used in NS-PSO, then, the particles move according to the neighborhood distance between the winner and them. The winner's neighborhood particles move toward the winner, so that they spread to whole space. For the particles which are not 1-neighbors of the winner but are connected near the winner, the gravitation toward the winner is strong. The other particles fly toward their *lbest*. In other words, the roles of the NS-PSO particles are determined by the connection relationship, and they produce the diversity of the particles. These effects avert the premature convergence, and the particles of NS-PSO can easily escape from the local optima.

Discussion about evaluation of each topology: Let us consider the network topology and its behavior in terms of average node-to-node distance L, which is also known as average shortest path length, and the average number of particles in local neighbor N_l. On 6×6 map, the average shortest path length L of respective topology; rectangular, hexagonal, cylinder and toroidal, are 6.6, 5.34, 5.8 and 5.0, respectively. The average number of particles N_l in local neighbor including itself of respective topology; rectangular, hexagonal, cylinder and toroidal, are 4.6, 6.18, 4.8 and 5.0. Because the cylinder and toroidal are the circular topology, the individuality of each particle is almost same. In other words, on toroidal-topology, L and N_l is completely same for any of the particles. Furthermore, L of toroidal-topology is the smallest in four NS-PSOs. From these effects, it is easy to transmit the information of *lbest* to the whole particles, therefore, the circular topology is effective for the unimodal function which is simple. However, the premature communication produces the premature convergence, then, toroidal-topology easily goes into local optima in the multimodal functions. On the other hand, NS-PSO with hexagonal-topology contains various kinds of particles which has different shortest path length and different size of local neighbors, although L is small and N_l is big. Because these effects produce the diversity of the particles and avert the premature convergence, the particles of NS-PSO with hexagonal-topology can easily escape from the local optima.

4 Conclusions

In this study, we have proposed Network-Structured Particle Swarm Optimizer (NS-PSO) with various neighborhood topology which is a collaboration between

Self-Organizing Map (SOM) and PSO. All particles of NS-PSO are connected to adjacent particles by a neighborhood relation, and their information are updated by the neighborhood topology. We have applied NS-PSO with various topology to optimization problems. and have confirmed that PSO, which has the specific network-structure, is more effective than the standard PSO, which has no neighborhood relationship. Furthermore, we have found that the toroidal-topology and the hexagonal-topology are suitable for the unimodal and for the multimodal function, respectively.

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