Synchronization Phenomena of van der Pol Oscillators Coupled by a Simplified Memristor

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***Abstract*—This study is about the synchronization phenomena of van der Pol oscillators coupled by a simplified memristor. The case connected with a memristor and the case connected with a pure resistor will be compared. From the results obtained, we will Observe the impact of connecting with the linear memristor.**

I. Introduction

Memristors are still in the development stage and are difficult to put into practical use [1].Memristors are studied assuming a variety of properties[2]-[4]. Elucidating the characteristics of memristors will contribute to future technological development [5]. A memristor is an element whose resistance changes in accordance with the current flowing through it or the magnetic flux penetrating it.

This research is not a study of memristors with specific characteristics, however rather a study to verify the phenomenon that memristors in general affect the synchronization phenomenon of oscillators. Therefore, the characteristics of the memristor are assumed to be relatively simple: the characteristics change in proportion to the magnetic flux penetrating the memristor. Hereafter, the memristor used in the circuit will be referred to as a "linear memristor". In order to show the influence of the memristor more clearly, a simulation of a circuit in which the linear memristor is replaced by a pure resistor is also performed and compared.

II. Proposed Model

Figure 1 shows the memristor model. The current and voltage of the memristor are denoted by i and v, and Ohm's law is described in Eq. (1).

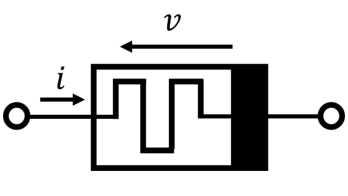

Fig. 1. Memristor model.

$$i = W(\varphi)v \tag{1}$$

The definition of memductance $W(\varphi)$ is assumed to be described in Eq. (2) with α as the proportionality constant.

$$W(\varphi) = \alpha\varphi \tag{2}$$

It is called a linear memristor because the memductance increases monotonically in proportion to the magnetic flux.

Figure 2 shows the circuit model. This model consists of two van der Pol oscillators coupled by the memristor.

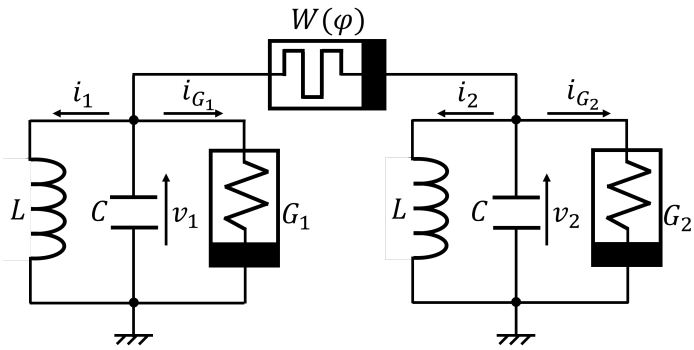

Fig. 2. Circuit Model.

Where the currents of the nonlinear resistor are denoted by i_{G1} and i_{G2}, and the current-voltage characteristics are described in Eq. (3).

$$i_{Gn} = -g_1 v_n + g_3 v_n^3 \quad (g_1, g_3 > 0) \quad (n = 1, 2) \tag{3}$$

The circuit equations are normalized by the following normalization parameters.

$$\begin{gathered} v_n = \sqrt{\frac{g_1}{g_3}}x_n, \quad i_n = \sqrt{\frac{g_1 C}{g_3 L}}y_n \quad (n = 1, 2) \\ \varepsilon = g_1\sqrt{\frac{L}{C}}, \quad \zeta = \alpha L\sqrt{\frac{g_1}{g_3}}, \quad t = \sqrt{LC}\tau \end{gathered} \tag{4}$$

From these equations, we see that the parameter for nonlinearity of the van der Pol oscillator is ε and the parameter for the memristor is ζ. The normalized circuit equations are shown in Eq. (6).

$$
\begin{cases}
\dfrac{dx_1}{d\tau} = \varepsilon x_1(1 - x_1^2) - \zeta(x_1 - x_2)(y_1 - y_2) - y_1 \\[2ex]
\dfrac{dx_2}{d\tau} = \varepsilon x_2(1 - x_2^2) + \zeta(x_1 - x_2)(y_1 - y_2) - y_2 \\[2ex]
\dfrac{dy_1}{d\tau} = x_1 \\[2ex]
\dfrac{dy_2}{d\tau} = x_2
\end{cases}
\tag{5}
$$

In the simulation, ε takes different values for each van der Pol oscillator. So ε for the oscillator on the left side of the circuit model is ε_1, and ε on the other side is ε_2.

III. Simulation Results

Simulation are performed by changing the four initial values. The simulation operates using the Runge-Kutta method. We compare the results when coupled by the linear memristor with the results when coupled by a pure resistor. Two different results are shown for each section under different conditions. Also, each result includes (i) a circuit attractor (x_2-y_2) and (ii) a phase difference (x_1-x_2).

We compare simulation results when initial values are changed without changing ε and ζ. Table I shows list of used conditions.

TABLE I
USED CONDITIONS.

	x_1	y_1	x_2	y_2	ε_1	ε_2	ζ
Condition 1	1.1	2.1	1.2	2.3	0.1	0.1	0.24
Condition 2	1.1	-2.1	-1.2	2.3	0.1	0.1	0.24

1) Pure Resistor:

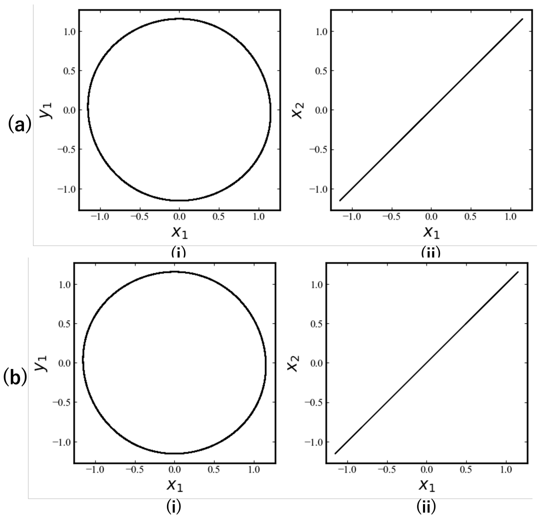

Fig. 3. Simulation result using condition 2 in Tab. I with pure resistor. (a) condition 1, (b) condition 2.

When connected with pure resistors, there is no difference in synchronization between the different initial values.

2) Linear Memristor:

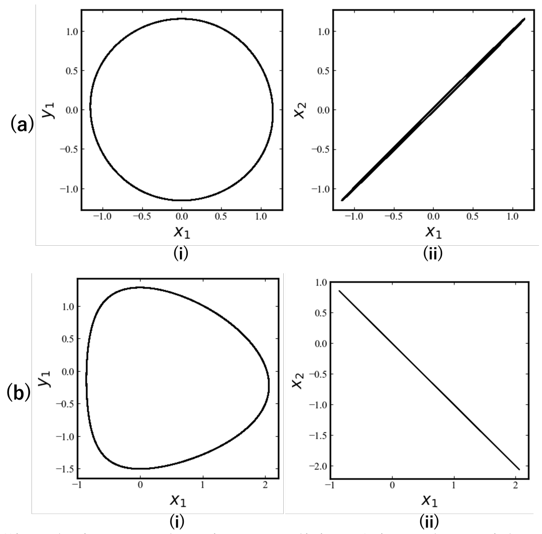

Fig. 4. Simulation result using condition 2 in Tab. I with Liniear memristor. (a) condition 1, (b) condition 2.

When connected with memristors, the attractors are distorted by the difference in initial values, and the coexistence of in-phase synchronization and anti-phase synchronization is observed.

IV. Conclusions

In this study, we observed the synchronization phenomena of van der Pol oscillators connected by memristors. Clearly different synchronization phenomena, in-phase synchronization and antiphase synchronization, are observed depending on the difference in initial values. In conclusion, the linear memristor has some effect on the synchronization phenomena of the van der Pol oscillator.

In the future, we aim to corroborate the output results with a theoretical analysis. Specifically, we aim to calculate the range of initial values at which synchronization phenomena occur for in-phase synchronization and anti-phase synchronization, respectively, using the averaging method.Also, we aim to observe changes in the attractor and Lissajous diagram by changing initial values ε or ζ over time.

References

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