Effect of Bridge on Synchronized Chaotic Circuit Networks with Different Chaos Parameters

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***Abstract*—It is investigated the collapse of synchronization when a bridge connection is added to two synchronized networks. It is also checked the effect of circuit networks with different dynamics on each other's networks. As a result, it was confirmed that the addition of the bridge connection caused a collapse in synchronization. It was also found that networks with more complex dynamics had a larger collapse of synchronization.**

I. Introduction

Chaotic circuit is nonlinear system that can generate diverse and complex dynamics due to their characteristics such as initial value sensitivities and nonlinearity, and their properties have attracted attention in many fields. In particular, the synchronization phenomenon of coupled chaotic circuits is used to analyze the behavior of complex and dynamic systems, such as models of the brain and nervous system in bioengineering [1] and signal synchronization in communication networks [2].

Network bridges function as connection between two networks and play an important role in information propagation and data relay between networks. Such network bridges have been researched in various fields, such as synchronization of chaotic circuit networks [3] and facilitating information distribution in social networks [4].

In this study, it is investigated how adding network bridges to two different networks changes the synchronization phenomena between the two networks. The effect of chaos parameters α on synchronization phenomena is confirmed when bridging two networks.

II. Network Model

Figure 1 shows the chaotic circuit used in this study. The chaotic circuit consists of a negative resistor, two capacitors, an inductor, and a bidirectional diode. The chaotic circuit is called the Mori-Shinriki circuit [5], [6].

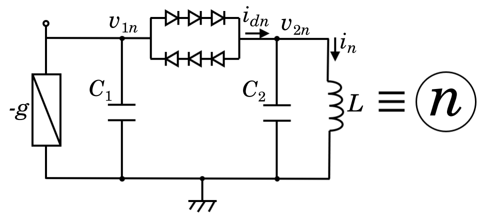


Fig. 1. Circuit model.

The circuit equations of the chaotic circuit are shown as follows:

$$\begin{cases} L\dfrac{di_n}{dt} = v_{2n} \\ C_1\dfrac{dv_{1n}}{dt} = gv_{1n} - i_{dn} - \dfrac{1}{R}\displaystyle\sum_{k\in S_n}(v_{1n} - v_{1k}) \\ C_2\dfrac{dv_{2n}}{dt} = -i_n + i_{dn}, \end{cases} \qquad (n = 1, 2, \cdots, 200) \tag{1}$$

S_n is the number of edges connected to the nth circuit. The $i-v$ characteristics of the nonlinear resistor consisting of diodes are described as follows:

$$i_{dn} = \begin{cases} G_d(v_{1n} - v_{2n} - a) & (v_{1n} - v_{2n} > a) \\ 0 & (|v_{1n} - v_{2n}| \le a) \\ G_d(v_{1n} - v_{2n} + a) & (v_{1n} - v_{2n} < -a). \end{cases} \tag{2}$$

By using the following parameters and variables:

$$i_n = \sqrt{\frac{C_2}{L}}ax_n,\ v_{1n} = ay_n,\ v_{2n} = az_n$$
$$t = \sqrt{LC_2}\tau,\ \text{“}\cdot\text{”} = \frac{d}{d\tau},\ \alpha = \frac{C_2}{C_1}$$
$$\beta = \sqrt{\frac{L}{C_2}}G_d,\ \gamma = \sqrt{\frac{L}{C_2}}g,\ \delta = \frac{1}{R}\sqrt{\frac{L}{C_2}},$$

the normalized circuit equations are described as follows:

$$\begin{cases} \dot{x_n} = z_n \\ \dot{y_n} = \alpha\gamma y_n - \alpha\beta f(y_n - z_n) - \alpha\delta\displaystyle\sum_{k\in S_n} f(y_n - y_k) \\ \dot{z_n} = \beta f(y_n - z_n) - x_n. \end{cases} \tag{3}$$

Where the nonlinear function corresponding to the $i-v$ characteristics in Eq. (2) is described as follows:

$$f(y_n - z_n) = \begin{cases} y_n - z_n - 1 & (y_n - z_n > 1) \\ 0 & (|y_n - z_n| \le 1) \\ y_n - z_n + 1 & (y_n - z_n < -1). \end{cases} \tag{4}$$

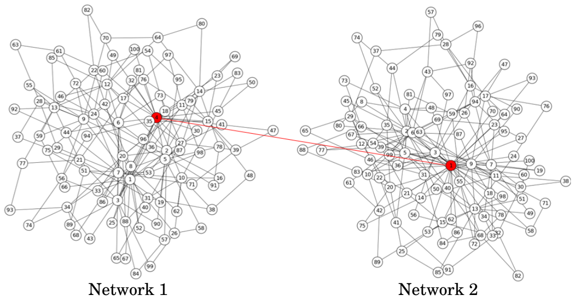
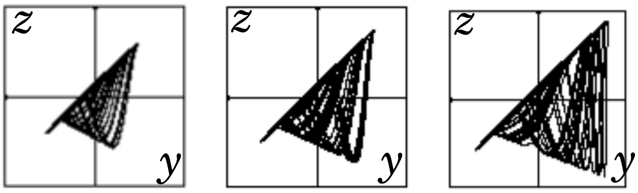


Fig. 2. Network model.



(a) (b) (c)

Fig. 3. Chaos attractors. (a) α = 0.40, (b) α = 0.50, (c) α = 0.60.

Figure 2 shows one of the models proposed in this study. Two networks with 100 circuits are constructed, and a bridge is added to connect the two networks. The red line in Fig. 2 shows the bridge. Each network is fully synchronized before adding the bridge. Bridge is added between highest degree nodes. α is called chaos parameter. It is prepared three patterns of networks with α of 0.40, 0.50 and 0.60. The chaos attractors at each α are shown in Fig. 3.

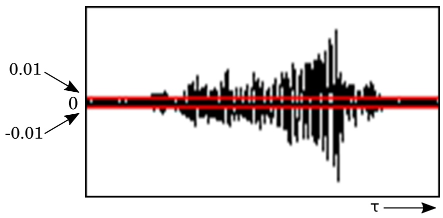


Fig. 4. Voltage difference waveform.

$$|y_i - y_j| \leq 0.01 \quad (i \neq j) \tag{5}$$

The synchronization rate is the percentage of the voltage difference between two circuits that is within the reference value. When the voltage difference is within the reference value, it is considered synchronous, and when it is not within the reference value, it is considered asynchronous. In this study, the reference value for synchronization is set at 0.01.

III. Simulation Results

The parameters on all circuits are fixed as $\beta = 20$ and $\gamma = 0.5$. α is changed at 0.40, 0.50 and 0.60. The coupling strength of network connections set as $\delta = 0.55$. The coupling strength of the bridge is varied from $\delta_b = 0.0$ to $\delta_b = 1.0$ in steps of 0.1. In this simulation, it is prepared four patterns of network combinations with different chaos parameters α: Network 1 - Network 2 = 0.50 - 0.50, 0.40 - 0.50, 0.50 - 0.60 and 0.40 - 0.60.

Table I shows the combinations of α and the average synchronization rate of each network and the average synchronization rate of the entire network when $\delta_b = 1.0$. Table I shows the between highest degree nodes. From Tab. I, it is confirmed the difference in the collapse of the synchronization for each network by connecting networks with different chaos parameters α. It is found that the networks with more complex dynamics have larger collapses in synchronization rate. It is also confirmed that the larger the difference in chaos parameters, the larger the difference in the average synchronization rate of each network.

TABLE I
Average synchronization rate
Bridge is highest degree nodes

	Network 1 [%]	Network 2 [%]	Entire Network [%]
0.50-0.50	57.4	58.0	57.7
0.40-0.50	72.6	57.4	65.0
0.50-0.60	57.0	48.3	52.6
0.40-0.60	71.6	48.2	59.2

IV. Conclusions

In this study, it was investigated the effect of chaos parameters on the synchronization of the networks when they are connected to each other. As a result, by connecting networks with different chaos parameter α, it was confirmed the difference in the collapse of the synchronization of each network, and it was found that networks with more complex dynamics have a larger collapse of the synchronization rate. In the future, we would like to increase the number of bridges and explore ways of adding bridges that can efficiently collapse the synchronization of the entire network.

References

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