Analysis of Synchronization States in Coupled Oscillators via Memristors as Ladder Structure

Yukinojo Kotani, Yoko Uwate and Yoshifumi Nishio
Dept. Electrical and Electronic Engineering
Tokushima University
Email: {kotani, uwate, nishio}@ee.tokushima-u.ac.jp

***Abstract*—A memristor has been focused on as a synapse in various studies. The interesting point of this circuit element is that resistance depends on charge and flux, so it has dynamics between current and voltage. Recently, many researchers have investigated synchronization in coupled circuits via memristors to apply for artificial network systems such as associative memory devices. Our research group has already proposed three coupled van der Pol oscillators with memristor couplings as a ladder structure, and obtained anti– and in–phase synchronization state by using memristors as synapses. However, we have not investigated the cause of this phenomenon in detail. In this study, we make clear synchronization states obtained by using Poincaré method.**

I. Introduction

Synchronization is one of the interesting nonlinear phenomena in nature, and it has been investigated to apply for artificial systems such as associative memory devices [1], [2], microgrid network system [3], [4], and IoT systems [5], [6]. Therefore, it is important for human lives to investigate synchronization phenomena. Synchronization was also obtained in coupled circuits. Recently, a memristor has been focused on as a synapse in coupled circuits because it has dynamics between charge and flux.

A memristor is the fourth basic circuit elements; a resistor, a capacitor and an inductor. It was mathematically introduced by L. O. Chua in 1971 [7], and it was developed using by Hewlett-Packard Lab in 2008 [8]. The interesting point of this circuit element is that resistance depends on charge and flux, so it has dynamics between current and voltage because charge and flux is defined as integral of current and voltage respectively. M. Itoh has proposed the memristor model characterized by piece–wise linear function [9]. This memristor model has been also used as synapses in coupled FitzHugh–Nagumo circuits [10].

In this study, we investigate synchronization in three coupled van der Pol oscillators with memristor couplings as a ladder structure in detail. We visualize synchronization states by using Poincaré methods.

II. Proposed model

Our memristor model is shown in Fig. 1 [9]. Figure 1 (a) shows a schematic model of a memristor. Resistance of the memristor is called memristance $M(q)$. Memristance is defined as the gradient of q - φ characteristic curve defined as the piecewise-linear function $\varphi(q)$ in Fig. 1 (b). q and φ denote charge and flux respectively.

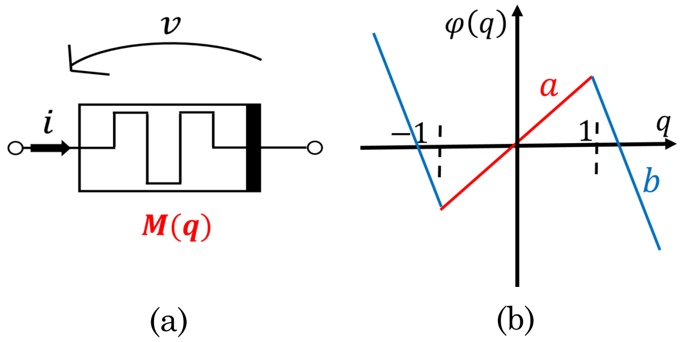

Fig. 1: Memristor model. (a) Schematic model. (b) $q-\varphi$ curve.

Hence, $\varphi(q)$ and $M(q)$ are represented as follows.

$$\varphi(q) = bq + 0.5(a-b)(|q+1| - |q-1|)$$
$$M(q) = \frac{d\varphi(q)}{dq} = \begin{cases} a & (|q| < 1) \\ b & (|q| > 1) \end{cases} \quad (1)$$
$$(a > 0,\ b < 0).$$

Figure 2 shows the three coupled van der Pol oscillators with memristor couplings as a ladder structure.

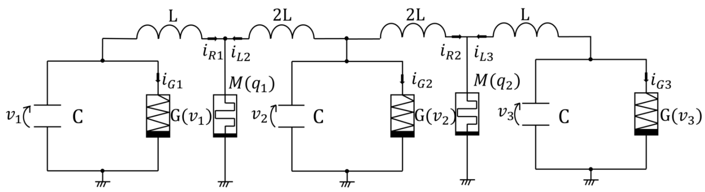

Fig. 2: Three coupled van der Pol oscillators with memristor couplings as a ladder structure.

A van der Pol oscillator consists of a third-power nonlinear resistor, a capacitor, and an inductor. The current–voltage characteristic curve of the nonlinear resistor is defined as Eq. (2).

$$i_{G,k} = -g_1 v_k + g_3 {v_k}^3 \quad (g_1,\ g_3 > 0). \quad (2)$$

Then, the circuit equations are described by the following equations.

$$\begin{cases} C\dfrac{dv_k}{dt} = -i_{R,k} - i_{L,k} - i_{G,k} \\ L\dfrac{di_{R,l}}{dt} = v_l - M(q_l)(i_{R,l} + i_{L,l+1}) \\ L\dfrac{di_{L,l}}{dt} = v_l - M(q_{l-1})(i_{R,l-1} + i_{L,l}) \\ 2L\dfrac{di_{R,2}}{dt} = v_2 - M(q_2)(i_{R,2} + i_{L,3}) \\ 2L\dfrac{di_{L,2}}{dt} = v_2 - M(q_1)(i_{R,1} + i_{L,2}) \\ \dfrac{dq_k}{dt} = i_{R,k} + i_{L,k+1} \\ M(q_k) = \dfrac{d\varphi(q_k)}{dq_k} = \begin{cases} a & (|q_k| < 1) \\ b & (|q_k| > 1) \end{cases} \\ \varphi(q_k) = bq_k + 0.5(a-b)(|q_k+1| - |q_k-1|). \end{cases} \tag{3}$$

These equations should be normalized to investigate the synchronization states for numerical calculation. By changing the variables and parameters such that

$$v_k = \sqrt{\frac{g_1}{g_3}}x_k,\ i_{R,k} = \sqrt{\frac{g_1 C}{g_3 L}}y_{R,k},\ i_{L,k} = \sqrt{\frac{g_1 C}{g_3 L}}y_{L,k},$$
$$q_k = z_k,\ \gamma = \sqrt{\frac{C}{L}},\ t = \sqrt{LC}\tau,\ \varepsilon = g_1\sqrt{\frac{L}{C}},\ \zeta = C\sqrt{\frac{g_1}{g_3}}.$$

The normalized circuit equations are obtained by changing the variables and the parameters as Eq. (2).

$$\begin{cases} \dfrac{dx_k}{d\tau} = \varepsilon(1 - x_k^2)x_k - y_{R,k} - y_{L,k} \\ \dfrac{dy_{R,k}}{d\tau} = x_k - y_{R,k} - \gamma M(z_k)(y_{R,k} + y_{L,k+1}) \\ \dfrac{dy_{L,k}}{d\tau} = x_k - y_{L,k} - \gamma M(z_k)(y_{R,k-1} + y_{L,k}) \\ \dfrac{dy_{R,k}}{d\tau} = 0.5\{x_k - y_{R,k} - \gamma M(z_k)(y_{R,k} + y_{L,k+1})\} \\ \dfrac{dy_{R,k}}{d\tau} = 0.5\{x_k - y_{R,k} - \gamma M(z_k)(y_{R,k} + y_{L,k+1})\} \\ \dfrac{dz_k}{d\tau} = \zeta(y_{R,k} + y_{L,k+1}) \\ M(z_k) = \dfrac{d\varphi(z_k)}{dz_k} = \begin{cases} a & (|z_k| < 1) \\ b & (|z_k| > 1) \end{cases} \\ \varphi(z_k) = bz_k + 0.5(a-b)(|z_k+1| - |z_k-1|). \end{cases} \tag{4}$$

Here, $k = 1, 2, 3$, $l = 1, 3$, $y_{L,1}$, $y_{R,3}$, $y_{R,0}$, $y_{L,0}$, $z_3 = 0$. τ is the scaling time, ε is the nonlinearity, γ is the coupling strength, and ζ is the coupling factor.

Our research group investigated synchronization phenomena in three coupled van der Pol oscillators with memristor couplings as a ladder structure [15]. Two synchronization states; only anti-phase synchronization between adjacent oscillators and in- and anti-phase synchronization between adjacent oscillators were obtained by setting different initial conditions. It means that these two phenomena coexist. Figure 3 shows the numerical results obtained by calculating Eq. (4).

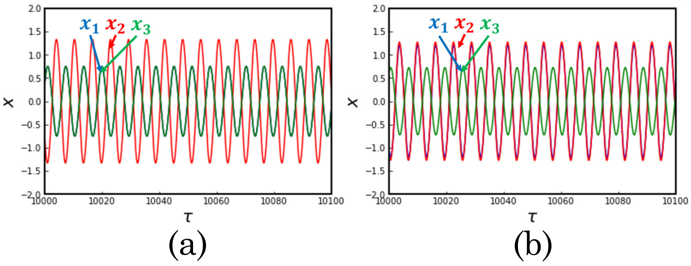

Fig. 3: Numerical results obtained by calculating Eq. (4). $a = 5.0$, $b = -0.1$, $\varepsilon = 0.1$, $\gamma = 0.1$, and $\zeta = 1.0$. (a) Only anti-phase synchronization, (b) Anti– and in–phase synchronization.

In Fig. 3 (a), first and second oscillators are anti–synchronized, and second and third oscillators are also anti–synchronized. This phenomenon has also obtained when resistors are used as synapses. In Fig. 3 (b), first and second oscillators are in–synchronized, and second and third oscillator are anti–synchronized. This phenomenon has newly confirmed by replacing resistors with memristors, so coexistence of these two phenomena is interesting in our proposed model.

III. Results

The normalized circuit equations are calculated by the Runge–Kutta method with step size $h = 0.01$, The parameters were set to $\varepsilon = 0.1$, $\gamma = 0.1$, $\zeta = 1.0$, $a = 5.0$, and $b = -0.1$.

First, we analyze the time–series of the relative phase differences $\Delta\theta_{1,k}$, x_k, M_k. We define Poincaré section as the region $H \in \{x_1 > 0,\ y_1 = 0\}$. $\Delta\theta_{1,k}$ is obtained by Eq. (5) when the solution orbits of first oscillator pass through the Poincaré section H.

$$\Delta\theta_{1,k} = |\theta_k - \theta_1| = |\arctan\frac{y_k}{x_k}| \tag{5}$$

Figure 3 (a) shows the numerical results obtained by calculating Eq. (4) and using Poinaré map when first and second oscillators were anti-synchronized. Figure 3 (b) also shows the numerical results when first and second oscillators were in-synchronized. Count means the times that the solution orbits pass through the Poincaré section H. Moreover, Tab. 1 shows the average of the absolute value of the relative phase differences.

TABLE I: Average relative phase differences.

Phase difference	Synchronization state	
	(a)	(b)
$\Delta\theta_{1,2}$	179.980	0.562
$\Delta\theta_{1,3}$	1.735	174.350

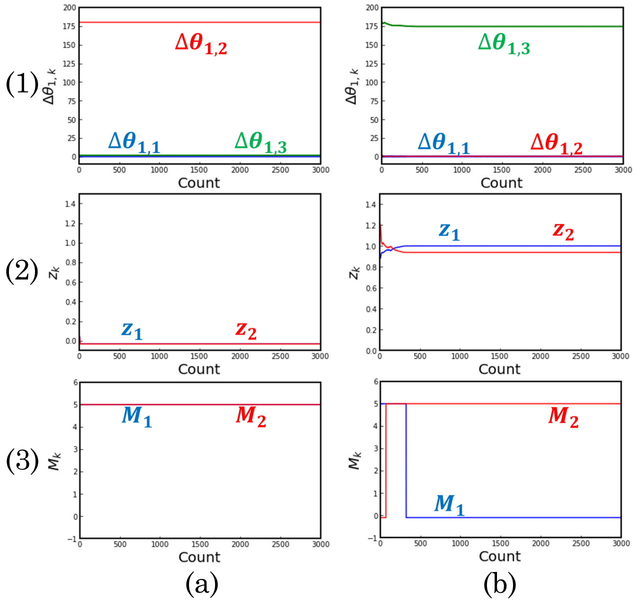

Fig. 4: Numerical results obtained by calculating Eq. (3) and using Poincaré map. (a) Only anti–phase synchronization, (b) Anti– and in–phase synchronization. (1) Time-series of $\Delta\theta_{1,k}$, (2) Time-series of z_k, (3) Time-series of M_k.

In case (a), z_k passing through the memristors converged to almost 0, and $M_k = a = 5.0$. All memristors behave as positive resistors, so only anti–phase synchronization between adjacent oscillators were confirmed. In case (b), z_1 passing through the memristors converged to value more than 1, and $M_1 = b = -0.1$. The memristor between first and second oscillators behaved as a negative resistor, so in–phase synchronization between first and second oscillators was confirmed.

IV. Conclusions

This study investigated synchronization phenomena in the coupled three van der Pol oscillators with memristor couplings as a ladder structure by Poincaré map. As a result, In addition, we made clear the changes of synchronization states by using Poincaré method. For the future works, we would like to use theoretical analysis and circuit simulations. These methods provide the fundamental evidences for the cause and effect relationship between the dynamics of the memristor couplings and the synchronization states.

References

[1] P. Maffezzoni, B. Bahr, Z. Zhang and L. Daniel, "Oscillator Array Models for Associative Memory and Pattern Recognition," *IEEE Trans. Circuits Syst. I.*, no. 6, pp. 1591-1598, Jun. 2015.

[2] P. Maffezzoni, B. Bahr, Z. Zhang and L. Daniel, "Analysis and Design of Boolean Associative Memories Made of Resonant Oscillator Arrays," *IEEE Trans. Circuits Syst. I.*, vol. 63, no. 11, pp. 1964-1973, Nov. 2016.

[3] J. Rocabert, A. Luna, F. Blaabjerg and P. Rodriguez, "Control of Power Converters in AC Microgrids," *IEEE Trans. on Ind. Electron.*, vol. 27, no. 11, pp. 4734-4749, Nov. 2012.

[4] M. Li, B. Wei, J. Matas, J. M. Guerrero and J. C. Vasquez, "Advanced Synchronization Control for Inverters Parallel Operation in Microgrids Using Coupled Hopf Oscillators," *CPSS Trans. Power Electron Appl.*, vol. 5, no. 3, pp. 224-234, Sep. 2020.

[5] A. A. Simiscuka, T. M. Markande, and G. Muntean,"Real-Virtual World Device Synchronization in a Cloud-Enabled Social Virtual Reality IoT Network," *IEEE Access*, vol. 7, pp. 106588–106599, Aug. 2019.

[6] Z. Idrees, J. Granados, Y. Sun, S. Latif, L. Gong, and Z. Zou, "IEEE 1588 for Clock Synchronization in Industrial IoT and Related Applications: A Review on Contributing Technologies, Protocols and Enhancement Methodologies," *IEEE Access*, vol. 8, pp. 155660–155678, Aug. 2020.

[7] L. O Chua, "Memristor-The Missing Circuit Element," *IEEE Trans. on Circuit Theory*, vol. CT-18, no. 5, pp. 507-519, Sep. 1971.

[8] D. B. Strukov, G. S. Snider, D. R. Stewart and R. S. Williams, "The missing memristor found," *Nature Lett.*, vol. 453, pp. 80-83, May 2008.

[9] M. Itoh and L. O. Chua, "Memristor Oscillators," *Int. J. Bifurcation Chaos*, vol. 18, no. 11, pp. 3183-3206, Sep. 2008.