Investigation of Average Synchronization Rate Transition with Topology Change in Networks Using van der Pol oscillators

Haruka Sakohira, Yoko Uwate, and Yoshifumi Nishio

Dept. of Electrical and Electronic Engineering, Tokushima University,
2-1 Minami-Josanjima, Tokushima, Japan
Email: {sakohira, uwate, nishio}@ee.tokushima-u.ac.jp

***Abstract*—Complex networks are everywhere around us and have attracted a lot of attentions. There have been many studies focusing on network topology, inter-node, and so on. In these studies, the network topologies have been fixed. Then, in this study, we investigate synchronization phenomena between nodes when the network topology is changed. As a result, they were confirmed that the synchronization phenomena change depending on the coupling strength.**

I. Introduction

Complex networks are around us and have attracted attention. They have properties such as scale-free, small-world, and clustered, and can be reproduced using nodes and edges. The Internet and transportation networks are familiar examples. Therefore, researches are conducted in various fields such as engineering, biology, economics, and so on [1]-[3]. Especially in engineering, investigations of synchronization between circuits have been conducted in recent years. Focusing on the topological structure of networks, research has been conducted on the phenomena of synchronization in different structures [4], [5]. However, the structure of real networks is rarely constant. Therefore, we construct networks using van der Pol oscillators and studied synchronization phenomena between nodes when the network topology is changed.

In this study, we constructed networks using 20 van der Pol oscillators. The network topologies are changed by removing edges that satisfy the threshold value (in this study, 100 % synchronization rate) and adding edges equal to the number of edges that are removed. Then we investigate how the synchronization rate between nodes changed. The results show that the synchronization rate between nodes tends to be higher or lower depending on the strength of the coupling.

II. System Model

A. Network Model

In this study, the Erdős Rényi model (ER model) is used to reproduce complex networks [6]. The model figure is shown in Fig. 1. The number of nodes is 20, and the coupling probability p is set to the average degree of 10. This model is one of a random network, and the cluster coefficients are small. It also has the property that the difference of the number of edges gathered at each node is small.

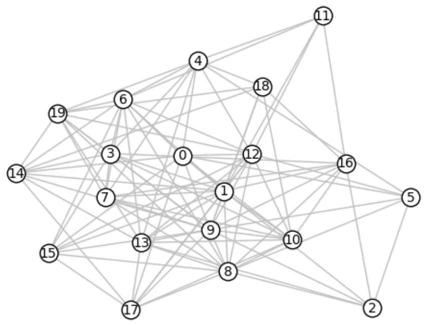


Fig. 1: ER model.

B. Circuit Model

In this study, the van der Pol oscillator is used to conduct simulations. The circuit is shown in Fig. 2. This circuit is simple, consisting only of an inductor, a capacitor, and a negative resistor. The network is constructed by connecting the circuits using resistors.

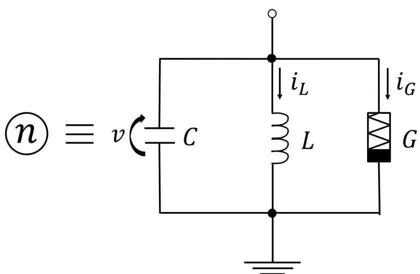


Fig. 2: van der Pol oscillator.

The circuit equation of this circuit is shown in Eq. (1).

$$\begin{cases} C\dfrac{dv_n}{dt} = -i_{L_n} - i_{g_n} - \displaystyle\sum_{k=1}^{20} \frac{1}{R}(v_n - v_k) \\ L\dfrac{di_n}{dt} = v_n \\ \qquad\qquad\qquad\qquad (n = 1, 2, \cdots, 20). \end{cases} \tag{1}$$

The equation of the nonlinear element is shown in Eq. (2).

$$i_g = -g_1 v + g_3 v^3. \tag{2}$$

Equation (1) is normalized using the following equations.

$$v = \sqrt{\frac{g_1}{g_3}}x,\ i = \sqrt{\frac{g_1 C}{g_3 L}}y,\ t = \sqrt{LC}\tau$$

$$\varepsilon = g_1\sqrt{\frac{L}{C}},\ \gamma = \frac{1}{R}\sqrt{\frac{L}{C}}.$$

The normalized equation is given in Eq. (3)

$$\begin{cases} \dfrac{dx_n}{d\tau} = \alpha\left\{\varepsilon x_n(1-x_n^2) - y_n - \displaystyle\sum_{k=1}^{20} E_{nk}\gamma(x_n - x_k)\right\} \\ \dfrac{dy_n}{d\tau} = x_n \\ \qquad\qquad\qquad\qquad (n=1,\ 2,\ \cdots,\ 20). \end{cases} \tag{3}$$

α is the micro error of each capacitor and E is the adjacency matrix of the network.

III. Simulation Results

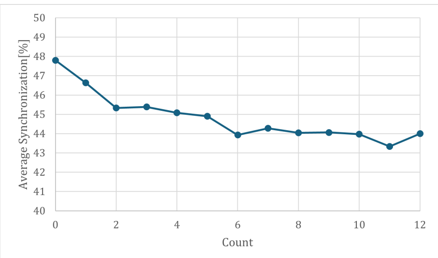
A. *Definition of synchronization*

In this study, the average synchronization rate is calculated from the synchronization rate when the voltage difference becomes steady state. The threshold value is set to Eq. (4). When the voltage difference is within this range, the state is the synchronized state, and when it is outside this range, the state is considered to be the unsynchronized state. n,k is the number of circuits.

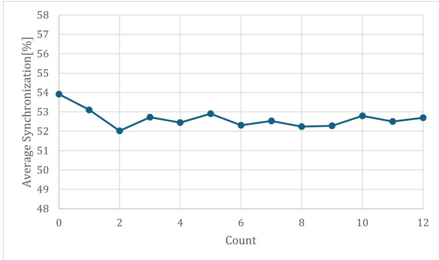
$$|x_n - x_k| < 0.01. \tag{4}$$

B. *Average Synchronization Rate*

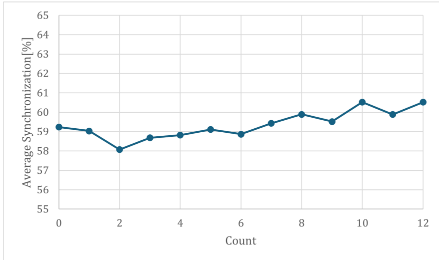
We investigate the change of synchronization rate between the circuits when the coupling strength is changed. The changes of the average synchronization rate when the coupling strength is changed in the range of γ = 0.1 - γ = 0.2 by γ = 0.02 are shown in Fig. 3. These average synchronization rates are the average of five times.



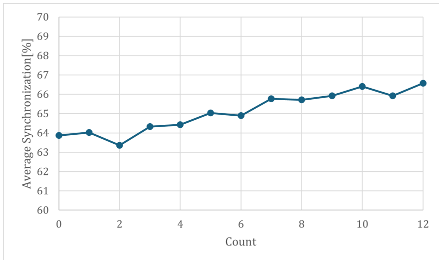
(a) γ=0.1.



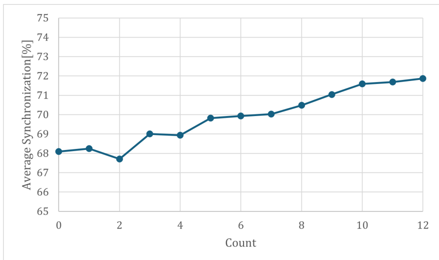
(b) γ=0.12.



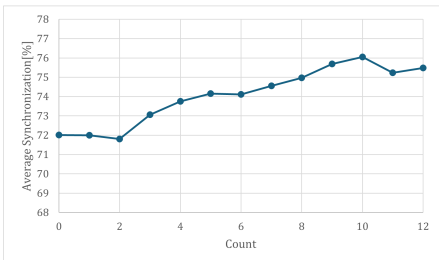
(c) γ=0.14.



(d) γ=0.16.



(e) γ=0.18.



(f) γ=0.2.

Fig. 3: Average Synchronization Rates

It can be seen that when all the synchronization rates are below γ = 0.12, the average synchronization rate tends to be lower. When the average synchronization rate is higher than γ = 0.14, the average synchronization rate tends to be higher. The synchronization rate tends to be higher when the coupling strength is stronger.

The number of edges per synchronization rate of the graph is closest to it out of the five averages. Figure 4 shows when γ = 0.1, when the synchronization rate tends to decrease, Fig. 5 shows when γ = 0.14, when it changes to increase, and Fig. 6 shows when γ = 0.18, when it tends to increase.

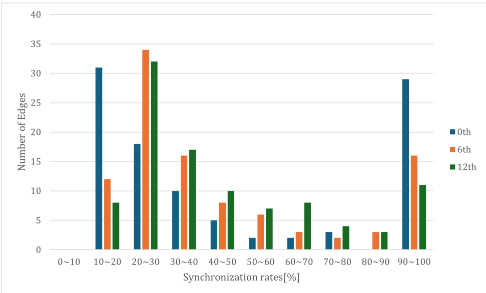


Fig. 4: Number of edges (γ=0.1).

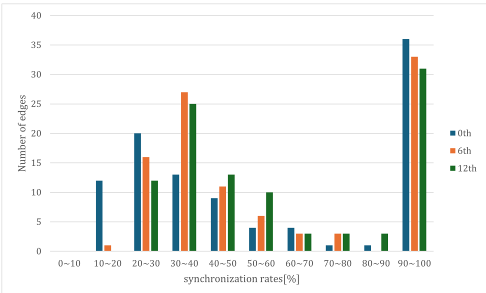


Fig. 5: Number of edges (γ=0.14).

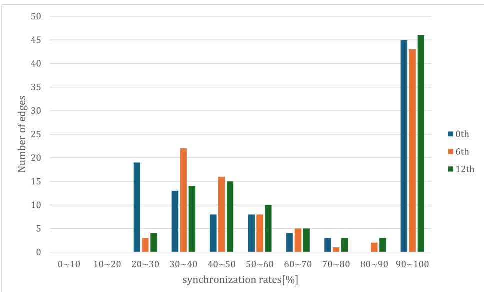


Fig. 6: Number of edges (γ=0.2).

It can be seen that the 90 - 100 % ratio tends to decrease. It also shows that the peaks tend to shift to higher synchronization rates.

IV. Conclusions

In this study, we investigated how the average synchronisation rate changes by changing the network topology. It was found that the average synchronisation rate changes between γ = 0.12 and γ = 0.14, where the average synchronisation rate becomes higher or lower. In addition, the synchronisation rate continues to decrease as the edges are continuously cut. Therefore, it is considered that the effect of the coupling strength when adding edges determines the change in the average synchronisation rate. In the future, we would like to investigate the effect of changes to the average synchronisation rate in more detail by increasing the number of trials. We would also like to conduct simulations that are closer to reality by extending the network.

References

[1] Charalampos Sergiou, Marios Lestas, Pavlos Antoniou, Christos Liaskos, Andreas Pitsillides, "Complex Systems: A Communication Networks Perspective Towards 6G", IEEE Access, vol. 8, pp. 89007-89030, March 2020.

[2] Tanujit Chakraborty Uttam Kumar, and J. Senthilnath, Swarup Chattopadhyay, Suchismita Das,(Senior Member, IEEE), "Searching for Heavy-Tailed Probability Distributions for Modeling Real-World Complex Networks" IEEE Access, Vol. 10, pp. 115092-115107, 2022

[3] Zhuang Ma And Guangdonghuang, "Image Recognition and Analysis: A Complex Network-Based Approach", IEEE Access, Vol.10, pp. 109537-109543, 2022.

[4] Jiajing Wu, Chi K. Tse, Francis C. M. Lau and Ivan W. H. Ho, "Analysis of Communication Network Performance From a Complex Network Perspective", IEEE Transactions on circuits and Systems-I: Regular Papers, vol. 60, No. 12, December 2013.

[5] Tsuyoshi Isozaki, Takumi Nara, Yoko Uwate and Yoshihumi Nishio, "Research on the Relationship between Network Structure and Synchronization Phenomena in Complex Networks by Using Oscillators", Proceedings of IEEE Workshop on Nonlinear Circuit Networks (NCN'20), pp.11-14, December 2020.

[6] D. Erdős and D. Rényi, "On the evolution of random graphs," Publ. Math. Inst. Hung. Acad. Sci, vol.5, pp. 17-60, 1960.