Coupled Oscillators with Sparse Coupling for Realization of Associative Memory

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***Abstract*—In this study, we applied sparse coupling to the coupling system of van der Pol oscillators and investigated the realization of associative memory. The phase difference of oscillator waveforms and the difference of output patterns by disconnecting the coupling between oscillators was clarified. As a result, it was confirmed that the realization of associative memory is possible using sparse coupling, however the maximum rate of coupling disconnection for which associative memory is possible depends on the combination of oscillators in the coupling system.**

I. Introduction

There has been research on synchronization phenomena observed from the coupling system of oscillators. It has been proven that coupling system of oscillators is able to realize pattern recognition and associative memory by using this phenomenon [1]. It is being studied with the goal of bringing associative memory capabilities closer to those of humans. Network sparsity has also been studied in CNNs to remove unnecessary computation and memory [2]. In previous studies, associative memory was achieved by using an associatron in the coupling system of oscillators. Furthermore, the relationship between the coupling strength between oscillators and associative memory was investigated [3]. When implementing the oscillator model in an actual circuit, there are many wires, which makes implementation difficult.

To reduce wiring due to loose coupling between oscillators, in this study, sparse coupling is used in the coupling system of oscillators to disconnect the coupling. The couplings to be disconnected are divided into three cases, and the phase difference and recall patterns are compared by the computer simulations. We also investigate the relationship between the rate of disconnection and associative memory.[1]

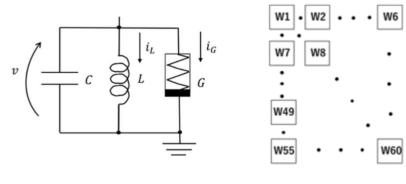
II. System Model of Associative memory using the coupling oscillators

Figure 1 (a) shows the circuit diagram of the van der Pol (VDP) oscillator. A model of a mutually coupled system consisting of 6 × 10 VDPs is shown in Fig. 1 (b). All oscillators are coupled to each other via resistors, and in this study, they are partially disconnected.

The normalized circuit equations are given as follows:

$$\begin{cases} \dfrac{dx_n}{d\tau} = \varepsilon x_n(1-x_n^2) - y_n + \alpha K \displaystyle\sum_{k=1}^{60}(x_n - x_k), \\ \dfrac{dy_n}{d\tau} = x_n \end{cases} \quad (1)$$

[1]The extended version of this study is being reviewed for ISCAS'24[4].



(a) (b)

Fig. 1. System model ((a) The circuit diagram of the VDP, (b) A model of a globally coupled system consisting.).

Where α is the connectivity matrix, it represents the state of coupling between oscillators, and it is possible to disconnect the coupling between the specified oscillators.

To realize associative memory, first synchronous simulation of the oscillator with the storage matrix for the input pattern is performed, and then the simulation is switched to simulation with it for the stored patterns. The output pattern is determined by the phase difference of the reference oscillator W1.

III. Simulation Results

The input pattern is shown in Fig. 3 (a) and The stored patterns are shown in Fig. 3 (b), (c) and (d).

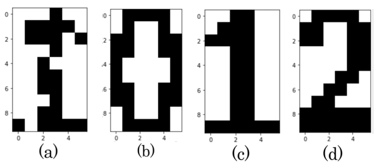


Fig. 2. 6 × 10 patterns ((a) is input pattern, (b), (c) and (d) are stored patterns.).

In the case of patterns in Fig. 2, the input pattern Fig. 2 (a) similar to the stored pattern Fig. 2 (c). If Fig. 2 (c) is a recall pattern, associative memory is considered to be realized. Equation (1) is calculated using Runge-Kutta method with the step size h = 0.1 for τ = 0 $\sim$ 2000. The initial values of x and y are random between 0 and 0.1, the parameters are ε = 0.1, s = 0.005. The storage matrix was changed from E_0 to E at τ = 100 for recalling a stored pattern. Count is the number of times the reference oscillator passed through the Poincar section.

A. Coupling Disconnection of Noisy Part of Oscillators

Oscillators in the noisy part of the model are those whose pixels have the wrong color when compared to the input pattern in Fig. 2 (a), and the correct stored pattern in Fig. 2 (c). Figure 3 shows the phase difference when the oscillator in the noisy part of Fig. 2 (a) is disconnected from the 38 randomly selected oscillators in Fig. 3 (a), and the 40 randomly selected oscillators in Fig. 3 (b). The results of the recall patterns at count = 300 are shown in Fig. 4.

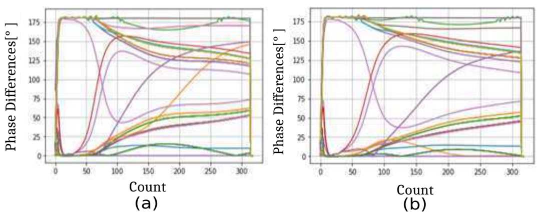


Fig. 3. Phase difference ((a) 8 × 38 disconnect coupling between oscillators, (b) 8 × 40 disconnect coupling between oscillators.).

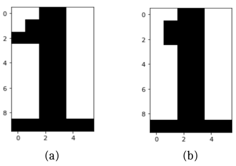


Fig. 4. Recall patterns ((a) 8 × 38 disconnect coupling between oscillators, (b) 8 × 40 disconnect coupling between oscillators.).

It can be seen from Fig. 2 (c) and Fig. 4 (b) that the color of W13 has changed from the stored pattern.

B. Coupling Disconnection of Randomly Selected Oscillators

The coupling between some randomly selected oscillators is disconnected from an equal number of other oscillators in this model. The phase difference of disconnected couplings of 29 randomly selected oscillators for each are shown in Fig. 5 (a), and 30 randomly selected oscillators are shown in Fig. 5 (b). The results of the recall patterns at count = 300 are shown in Fig. 6.

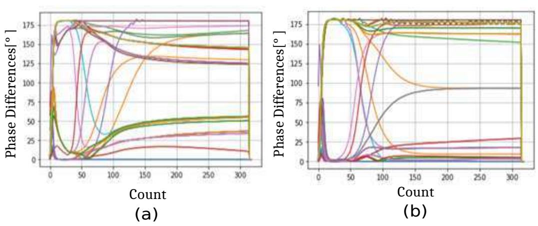


Fig. 5. Phase difference ((a) 29 × 29 disconnect coupling between oscillators, (b) 30 × 30 disconnect coupling between oscillators.).

The maximum value at which associative memory could be realized was 28 when the same number of oscillator connections were disconnected.

Finally, Add simulations using 8 randomly selected oscillators, summarize whether associative memory is realized in

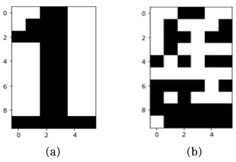


Fig. 6. Recall patterns ((a) 29 × 29 disconnect coupling between oscillators, (b) 30 × 30 disconnect coupling between oscillators.).

each of the simulation, the number of disconnections in the simulations and the rate of coupling disconnections in the model are shown in Table I.

TABLE I
THE RATE OF COUPLING DISCONNECTION

	The number of disconnections in simulation					
	8×38	***8×40***	***8×51***	***8×52***	***29×29***	***30×30***
The rate of disconnected couplings[%]	17.2	18.0	23.0	23.5	47.5	50.8
Realization of associative memory	○	×	○	×	○	×

The same number of disconnected oscillators can realize associative memory even though the rate of coupling disconnection is higher. Therefore, it can be observed that the larger the rate of coupling disconnection, the larger the recall pattern is broken.

IV. CONCLUSIONS

In this study, we proposed to use sparse coupling for the coupling system of van der Pol oscillators to disconnect the coupling. The realization of associative memory was investigated based on the phase difference of oscillation waveforms and recall patterns. As a result, it was possible to realize associative memory using sparse coupling, but associative memory was not realized when the rate of coupling disconnection was high. When the oscillator and the coupling of the noisy part were disconnected, the recall pattern was not broken and the coupling disconnection rate was small, however when the 30 × 30 couplings were disconnected, the recall pattern was broken significantly. In summary, it was possible to confirm that the rate of coupling disconnection varied depending on the combination of the 60 oscillators.

For the future work, we would like to investigate the couplings that can be preferentially disconnected when disconnecting couplings. In addition, a circuit simulation is to be performed to confirm the difference from this study.

REFERENCES

[1] P. Maffezzoni, B. Bahr, Z. Zhang, and L. Daniel, " Oscillator Array Models for Associative Memory and Pattern Recognition ", IEEE Transactions on Circuits and Systems I: Regular Papers, vol. 62, no. 6, pp. 1591–1598, June 2015.

[2] Y. Liang, L. Lu, and J. Xie, " OMNI: A Framework for Integrating Hardware and Software Optimizations for Sparse CNNs ", IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems, vol. 40, no. 8, pp. 1648-1661, August 2021.

[3] K. Nakano, "Associatron- A Model of Associative Memory", IEEE Transactions on Systems, Man, and Cybernetics. no. 3, pp. 380-388, July 1972.

[4] K. Fukuta, Y. Uwate, and Y. Nishio, "Associative Memory Function Using Coupled Oscillators with Sparse Coupling ", IEEE International Symposium on Circuits and Systems (ISCAS), 2024.(being submitted)