Analysis for Synchronization in Dependence on Memristor Behavior in Coupled Oscillators

Yukinojo Kotani, Yoko Uwate and Yoshifumi Nishio
Dept. Electrical and Electronic Eng.,
Tokushima University
Email: {kotani, uwate, nishio}@ee.tokushima-u.ac.jp

***Abstract*—In this study, we investigate synchronization phenomena in coupled oscillatory networks with the memristor couplings as a ring structure. We focus on the dynamics of a memristor, and our proposed method provides the dynamics for the couplings. This study confirmed that three different synchronization state types: multi-phase synchronization state, in-phase and anti-phase synchronization state and amplitude death were obtained by depending on the power consumption of the memristor couplings.**

I. Introduction

Synchronization phenomena observed from the coupled oscillatory networks have been studied. Many studies related to synchronization phenomena in oscillatory networks have important roles: analyzing the characteristics of networks [1]. Therefore, various synchronization phenomena in oscillatory networks need to be investigated.

A memristor is the fourth basic circuit elements; a resistor, a capacitor and an inductor. It is introduced by L. O. Chua in 1971 [2], and it was developed by Hewlett-Packard Lab in 2008 [3]. Resistance value of a memristor is characterized by a charge or a flux, whereas a resistor is characterized by a current or a voltage. The charge and the flux are defined as the integral of the current and the voltage respectively. The memristor has the dynamics compared to the resistor.

In previous studies, synchronization phenomena in the coupled oscillatory networks were investigated. The coupled oscillatory networks via resistors were proposed, and various synchronization phenomena were investigated [4]. In addition, the memristor coupling were proposed in several circuit models, and synchronization phenomena were investigated. Therefore, this study focuses on the memristor couplings in the oscillatory network as ring structures.

In this study, we focus on the memristor coupling compared to the conventional coupling methods of the other circuit elements depended on the current and the voltage. We investigate synchronization state types analyzing the time-series of the voltages, the memristors and the phase differences. In addition, we investigate the cause and effect relationship between synchronization state types and the dynamics of the memristor by calculating the power consumption of the memristor couplings. [1]

[1]The extended version of the study is being reviewed for ISCAS'24 [5].

II. Oscillatory Network Model

Figure 1 shows the Nth oscillatory network with the memristor couplings as a ring structure. r is the tiny resistors to avoid L-loop for calculating equations.

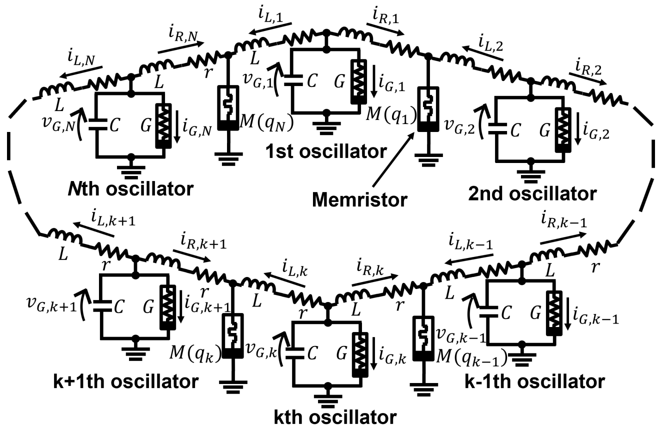

Fig. 1: Oscillatory network with memristor couplings as a ring structure.

Circuit equations are obtained from Kirchhoff's laws. These equations need to be normalized to investigate the synchronization states by using computer simulations.

The normalized circuit equations are obtained by changing the variables and the parameters as Eq. (1).

$$\begin{aligned}
\frac{dx_k}{d\tau} &= \varepsilon(1-x_k^2)x_k - y_{R,k} - y_{L,k}\\
\frac{dy_{R,k}}{d\tau} &= \frac{1}{2}(x_k - \eta y_{R,k} - M(z)(y_{R,k}+y_{L,k+1}))\\
\frac{dy_{L,k}}{d\tau} &= \frac{1}{2}(x_k - \eta y_{L,k} - M(z)(y_{R,k-1}+y_{L,k}))\\
\frac{dz_k}{dt} &= \zeta(y_{R,k}+y_{L,k+1})\\
M(z_k) &= \frac{d\varphi(z_k)}{dq} = \begin{cases} a & (|z_k|<1)\\ b & (|z_k|>1)\end{cases}\\
\varphi(z_k) &= bz_k + 0.5(a-b)(|z_k+1| - |z_k-1|).
\end{aligned} \tag{1}$$

where τ is the scaling time, ε is the nonlinearity, γ is the coupling strength, ζ is the coupling factor and η is the factor of the tiny resistor.

III. Results

The normalized circuit equations were calculated by the Runge-Kutta method with step size $h = 0.01$. The parameters were set to $\tau = 20,000$, $\varepsilon = 0.1$, $\gamma = 1$, $\zeta = 0.1$ and $\eta = 0.001$. The parameters of the memristors $a = 0.1$ and $b = 10$.

In this section, we analyzed the time-series of x, the time-sereis of M and the phase differences. The phase differences are calculated by using Poincaré maps. The methods of calculating the phase differences is described as follows.

1) A van der Pol oscillator with third-power has a stable limit cycle, so Poincaré section is defined as the plane that $x(\tau) > 0$ and $y(\tau) = 0$ in phase plane.
2) If x_1 and y_1 move from fourth quadrant to first quadrant ($x_1(\tau - 1) > 0$, $y_1(\tau - 1) < 0$ and $y_1(\tau) > 0$), each of the relative phase difference for first oscillator is calculated. Count is the number of the movement of the solutions x_1 and y_1 from fourth quadrant to first quadrant.

Figure 2 shows the simulation results. We confirmed three different synchronization state types: three-phase synchronization state, in-phase and anti-phase synchronization state and amplitude death.

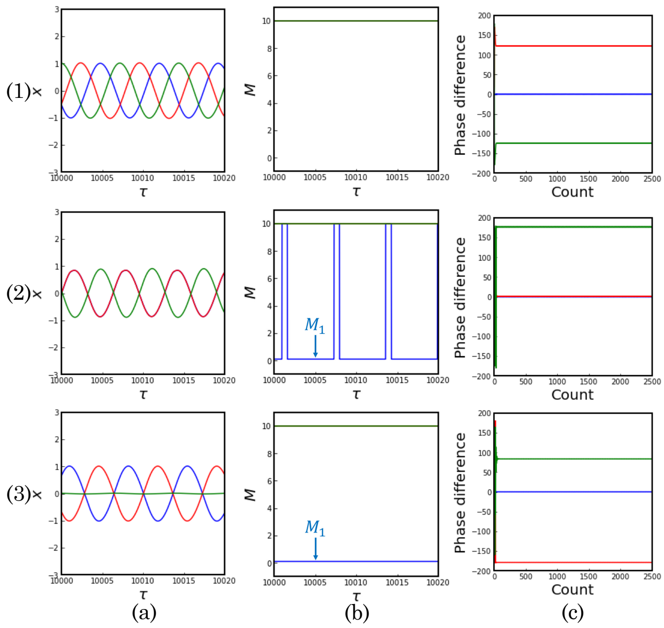

Fig. 2: Simulation results. (1): Three-phase, (2): In-phase and anti-phase, (3): Amplitude death, (a): Time-series of x, (b): Time-series of M, (c): Phase differences.

In these simulation results, for the case of (1), Nth-phase synchronization state were observed because all memristances assumed b constantly. For the cases of (2) and (3), in-phase and anti-phase synchronization state and amplitude death were confirmed because M_1 assumed a and the other memristances assumed b. Therefore, in-phase and anti-phase synchronization states and amplitude death were arisen from the dynamics of the memristor couplings.

Next, we investigated the average power consumption. The total of the average power consumption P_{all} in the oscillatory network with the memristor couplings can be calculated by Eq. (2). T is a long period. Therefore, Table I summarize the average power consumption compared Nth-phase synchronization, in-phase and anti-phase synchronizations and amplitude death.

$$P_{all} = \frac{1}{T} \sum_{k=1}^{T} \gamma M(z_k)(y_{R,k} + y_{L,k+1})^2. \tag{2}$$

TABLE I: Average Power Consumption ($N = 3$).

Power	Synchronization State Types		
	Three-phase	In-phase and Anti-phase	Amplitude Death
$P_{1,2}$	0.0128	0.0452	0.00000485
$P_{2,3}$	0.0125	0.0000585	0.0127
$P_{3,1}$	0.0128	0.0000574	0.0128
P_{all}	0.0382	0.0453	0.0255

In these simulations, for the case of Nth-phase synchronization, each of the average power consumption was the same values because all memristances equaled to b. For the case of in-phase and anti-phase synchronizations state, even number pairs of anti-phase components were canceled each other and the other component were not canceled, so the total average power consumption was much larger than the case of Nth-phase synchronization. However, for the case of amplitude death, even number pairs of anti-phase components were also canceled each other and the amplitude of the other component was dead, so the total average power consumption of the case of amplitude death was less than the case of Nth-phase synchronization. Therefore, our simulation results show that in-phase and anti-phase synchronizations state and amplitude death is caused by the dynamics of the memristor couplings.

IV. Conclusions

This study proposed the memristor couplings in coupled oscillatory networks as a ring structure. This study confirmed that three different synchronization state types: multi-phase synchronization state, in-phase and anti-phase synchronization state and amplitude death were obtained by depending on the power consumption of the memristor couplings.

References

[1] G. Yang, D. Tong, Q. Chen and W. Zhou,"Fixed-Time Synchronization and Energy Consumption for Kuramoto-Oscillator Networks With Multilayer Distributed Control", *IEEE Trans. Circuits Syst. II Regul. Pap*, vol. 70, no. 4, April 2023.
[2] L. O Chua, "Memristor-The Missing Circuit Element", *IEEE Trans. on Circuit Theory*, vol. CT-18, no. 5, pp. 507-519, Sep. 1971.
[3] D. B. Strukov, G. S. Snider, D. R. Stewart and R. S. Williams, "The missing memristor found", *Nature Lett.*, vol. 453, pp. 80-83, May 2008.
[4] Y. Setou, Y. Nishio and A. Ushida, "Synchronization Phenomena in Resistively Coupled Oscillators with Different Frequencies", *IEICE Trans. Fundam.*, vol. E79-A, no. 10, pp. 1575–1580, Oct. 1996.
[5] Y. Kotani, Y. Uwate and Y. Nishio, "Synchronizations in Oscillatory Networks with Memristor Couplings as Ring Structure", *Proc. of Int. Conf. ISCAS'24*, 2024. (being submitted)