Performance of Communication Network Focusing on Complex Networks Consisting of van der Pol Oscillators

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***Abstract*—Complex networks have been the subject of great attention because some characteristics of complex networks are closely related to the real-world network. In this study, we study the performance of communication networks from perspective of complex networks. We first investigate the synchronization between nodes in complex networks, and then relate the synchronization to communication network performance. Synchronization in communication networks is very important for the transmitting and receiving of information. If the network is congested, it may take a long time to send information, or the information you want to send may be lost in the process. Therefore we aim to improve the efficiency of information transmission.**

I. INTRODUCTION

Complex networks are modeled with nodes and edges based on several properties of real-world networks. As example, there are properties of small world and scale free. small world means that the distance between nodes is short compared to the total number of nodes. The second is the property of scale free. Scale free means that a small number of nodes, called hubs, have large degree, while almost all nodes have relatively small degree. Further in our daily lives, transportation networks and the Internet are examples of complex networks. Thus Complex networks have attracted a lot of attention from various fields such as sociology, biology and physics. In the field of engineering, the performance of communication networks has been studied with a focus on complex networks [1], [2]. Moreover the studies of synchronization between nodes in complex networks have identified some interesting phenomena [3].

In this study, first, we build two network models using van der Pol oscillators and calculate the synchronization rate between nodes. Next, we set parameters related to the communication network based on the synchronization rate and evaluate the performance.

II. NETWORK MODELS

In this study, we use two network models, Erdős Rényi model (ER random network) [4] and Barabási Albert model (BA scale free network) [5]. Each of these network models has some properties.

First, Fig. 1 shows the ER random network with 100 nodes. This network has properties such as the degree distribution follows the binomial distribution and short distance between nodes. Here, following the binomial distribution means that the number of nodes with degrees close to the average degree is larger.

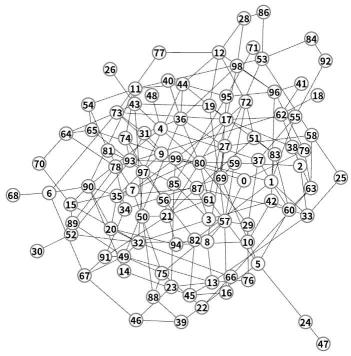


Fig. 1. ER random network.

Second, Fig. 2 shows the BA scale free network with 100 nodes. This network has properties such as the degree distribution follows the power law, short distance between nodes and growth and priority selection. Here, following the power law distribution means that a few nodes are connected to many other nodes and have a large degree. On the other hand, most other nodes are connected to only a few nodes and have a small degree. In addition, priority selection means that newly added nodes are more likely to be connected with nodes of higher degree during formation of the network.

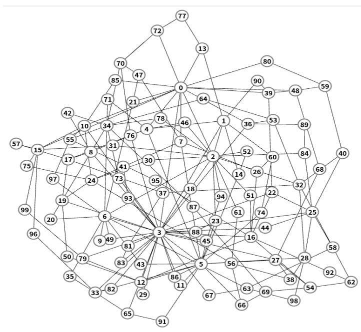


Fig. 2. BA scale free network.

The conditions of the networks created in this case are shown in Table I. The number of nodes N, average node degree d_{avg}, maximum node degree d_{max}, minimum node degree d_{min} and average distance D_{avg} of two network models.

TABLE I
THE CONDITIONS OF THE NETWORKS.

network	ER random	BA scale free
N	100	100
d_{avg}	4.08	3.92
d_{max}	9	32
d_{min}	1	2
D_{avg}	3.38	2.98

In this study, we set the average node degree to be close to 4.0 each network model. Further as properties of the network, the maximum node degree of the BA scale free network is larger than that of the ER random network and the average distance of the BA scale free network is shorter than that of the ER random network.

III. SYSTEM MODEL

Figure 3 shows van der Pol oscillator. This oscillator consists of a capacitor, an inductor and a nonlinear element. Further one circuit is considered to be a single node and a network is formed by connecting circuits to each other using resistors. In this study, 100 van der Pol oscillators are connected to form each network.

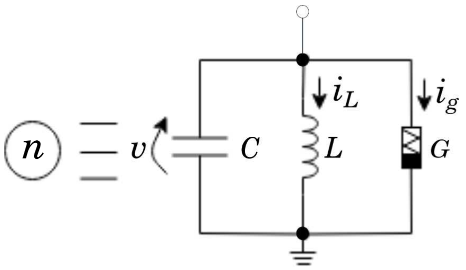


Fig. 3. van der Pol oscillator.

When the circuits are connected by using resistors, the circuit equations are given as follows:

$$\begin{cases} C\dfrac{dv_n}{dt} = -i_{L(n)} - i_{g(n)} - \displaystyle\sum_{n,k=1}^{100} \frac{1}{R}(v_n - v_k) \\ L\dfrac{di_n}{dt} = v_n \\ \qquad\qquad (n, k = 1, 2, \cdots, 100), \end{cases} \tag{1}$$

where the parameter i_g is the equation for the nonlinear element and described as follows:

$$i_g = -g_1 v + g_3 v^3. \tag{2}$$

By using the parameters and the variables:

$$v = \sqrt{\frac{g_1}{g_3}}x,\ i = \sqrt{\frac{g_1 C}{g_3 L}}y,\ t = \sqrt{LC}\tau$$

$$\varepsilon = g_1\sqrt{\frac{L}{C}},\ \gamma = \frac{1}{R}\sqrt{\frac{L}{C}}.$$

The normalized circuit equations are described as follows:

$$\begin{cases} \dfrac{dx_n}{d\tau} = \alpha\left\{\varepsilon x_n(1 - x_n^2) - y_n - \displaystyle\sum_{n,k=1}^{100} E_{nk}\gamma_{nk}(x_n - x_k)\right\} \\ \dfrac{dy_n}{d\tau} = x_n \\ \qquad\qquad (n, k = 1, 2, \cdots, 100), \end{cases} \tag{3}$$

where E_{nk} represents the connected state of node n and node k. If node n and node k are connected, $E_{nk} = 1$ and if they are not connected, $E_{nk} = 0$. Further we set the parameters of van der Pol oscillator as $\varepsilon = 0.1$ and $\alpha = 1.0$. Here, α represents the small error on the capacitor, given randomly in the [0.95:1.05] range for each circuit.

A. Synchronization

In this study, we investigate the synchronization between nodes in the network and calculate it as a percentage in the two network models.

In order to analyze synchronization, we define the synchronization as following equation.

$$|x_n - x_k| < 0.01, \tag{4}$$

where n and k are number of circuits. Figure 4 shows the differential voltage waveform observed in this study. The two lines in Fig. 4 represent the thresholds shown in Eq. (4). If the voltage difference falls within these thresholds, it is considered synchronous. When calculating the synchronization rates, we use the voltage differences in the range of 10000 to 20000 in τ to account for the time it takes to settle to a steady state.

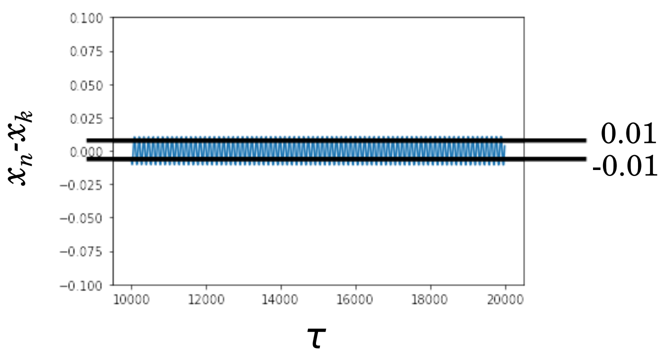


Fig. 4. The differential voltage waveform.

IV. COMMUNICATION NETWORK OPERATION

In this study, information is represented as packets. Moreover we relate oscillator synchronization to communication network performance.

A. Packets transmission algorithm

Packets are generated by the host nodes and transmit through the links one hop at a step until they reach the destinations. Moreover each node in the network has a buffer, the buffer size for node n being $B(n)$. Then, the packets transmission algorithm operates as follows:

1) Packets Generation
   At each time step, hosts generate R packets.
2) Packets Transmission
   The number of transmitting packets for node per step is S. At each time step, packets of each node are forwarded to their destinations by one step according to the routing algorithm.
3) Packets Dropped
   If the total number of packets reaching one node is larger than buffer size $B(n)$, transmitted packets are dropped.
4) Packets Released
   Packets already arrived at their destinations are released from the buffer.

Further $B(n) = \mu k(n)$, where $k(n)$ is degree of the nth node and μ is control parameter. The number of transmitting packets S is set three different patterns. The first is set randomly in the range of 1 to 10. The second is set to constant (here S = 5). The third is set based on the synchronization rate.

B. Routing algorithms

In this study, we study the performance of the network under two different algorithms, shortest path (SP) routing and high synchronization (HS) routing. A shortest path refers to the path with minimum hops from the source to the destination. HS routing selects the path with the highest synchronization rate between nodes.

C. Performance parameter

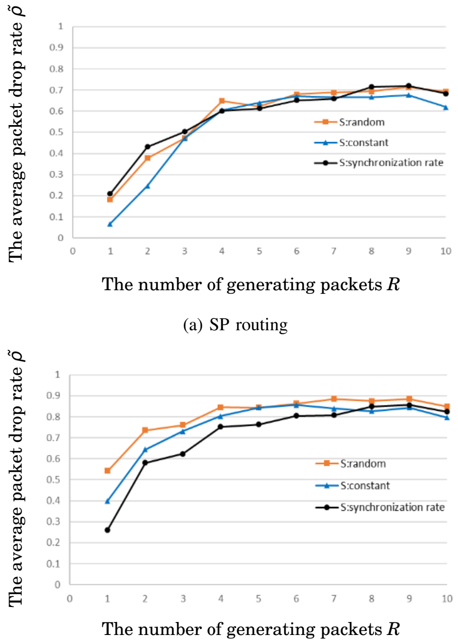
We define a performance parameter ρ to compare the network performance. ρ is packet drop rate and it is described the equation as follows:

$$\rho = \frac{The\ number\ of\ dropped\ packets\ in\ time\ step}{The\ number\ of\ generated\ packets\ in\ time\ step}. \quad (5)$$

V. RESULTS

We show the results of the relation between average packet drop rates $\tilde{\rho}$ and the number of generating packets R.

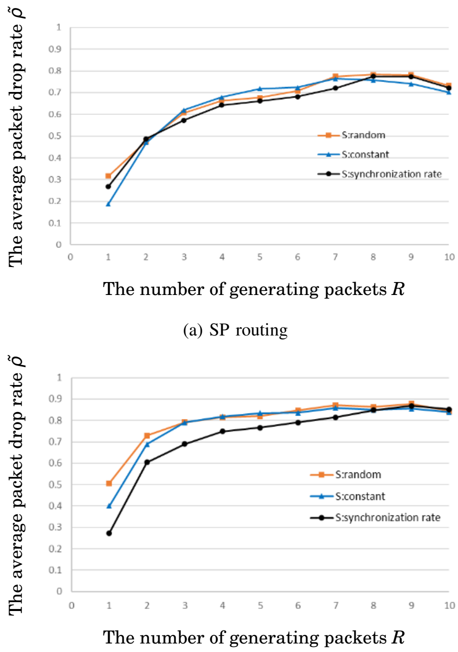
First, we compare S under two routing algorithms in the ER random network and the BA scale free network. Figure 5 shows the results of three different patterns of the number of transmitting packets S in the ER random network. Further Fig. 5(a) shows the results under the SP routing and Fig. 5(b) shows the results under the HS routing. Figure 6 shows the results of three different patterns of the number of transmitting packets S in the BA scale free network. Further Fig. 6(a) shows the results under the SP routing and Fig. 6(b) shows the results under the HS routing.



(a) SP routing

(b) HS routing

Fig. 5. Three different patterns of the number of transmitting packets S (ER random network).



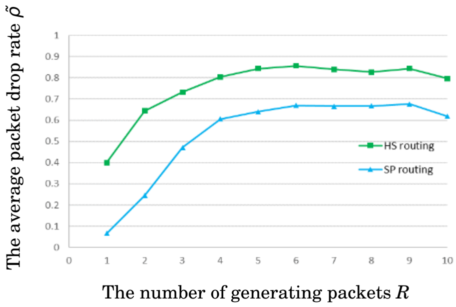
(a) SP routing

(b) HS routing

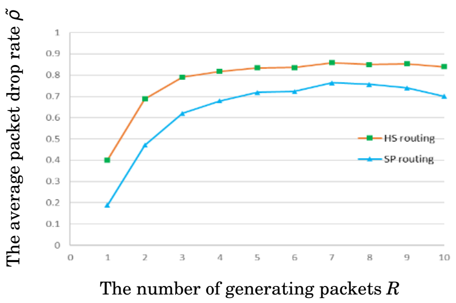
Fig. 6. Three different patterns of the number of transmitting packets S (BA scale free network).

As a result, we can see from Fig. 5(a) and Fig. 6(a) that in the value of R is small such as 1 or 2, when S is set to constant, $\tilde{\rho}$ is lower than the others S. However for values of R other than 1 or 2, $\tilde{\rho}$ is almost the same and has little to do with the difference in the S settings. Further it can been seen from Fig. 5(b) and Fig. 6(b) that when S is set based on the synchronization rate, $\tilde{\rho}$ is lower than the others S. In this simulation, the average S of the three patterns is set to be almost the same. However, when S is set based on the synchronization rate under the HS routing, the average S becomes larger than the others. It is considered that this has something to do with it. Therefore it can been seen that when the average S is set to be the same, there is not much difference depending on how the S is set.

Next, we compare SP routing with HS routing for each network. In this case, S is set to constant. Figure 7 shows the results for two different routing algorithms, and Fig. 7(a) for the ER random network and Fig. 7(b) for the BA scale free network.



(a) ER random network

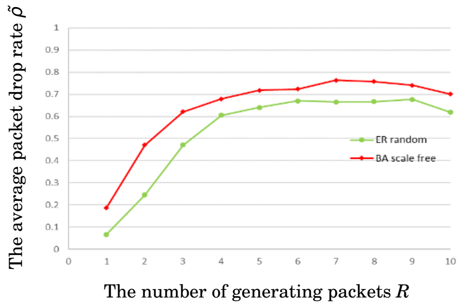


(b) BA scale free network

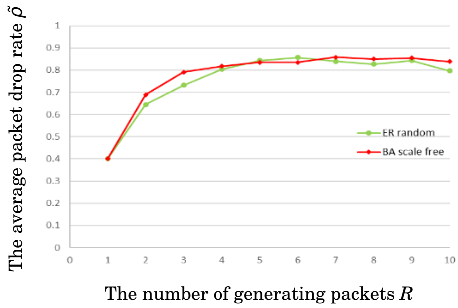
Fig. 7. Comparing the routing algorithms in the ER random network and the BA scale free network.

From Fig. 7, we can see that $\tilde{\rho}$ is lower under the SP routing in both networks. Therefore it can been seen that the routing algorithm affects the network performance.

Finally, we compare the ER random network with the BA scale free network under two routing algorithms. In this case, S is set to constant. Figure 8 shows the results of comparing the two networks under the same routing algorithm, and Fig. 8(a) shows the result under the SP routing and Fig. 8(b) shows the result under the HS routing.



(a) SP routing



(b) HS routing

Fig. 8. Comparing the networks under the SP routing and the HS routing.

From Fig. 8(a), we can see that $\tilde{\rho}$ is lower for the ER random under the SP routing and from Fig. 8(b), it can be seen that there is almost no difference under the HS routing. Therefore it can been seen that the network structure affects the network performance under certain routing algorithms.

VI. Conclusion

In this study, we first investigated the synchronization between nodes in complex networks, and then investigated the network performance by relating the synchronization to the efficiency of information transmission. It was confirmed that when the average S is set to be the same, there is not much difference depending on how the S is set. Further it is confirmed that the HS routing is inferior to the SP routing. It is considered that this is due to the fact that the same node is selected multiple times as a transit point under the HS routing. We would like to study on improving the HS routing.

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