Correlation between Chaos Propagation and Features of Complex Networks

Naoto Yonemoto†, Kenta Ago‡, Yoko Uwate† and Yoshifumi Nishio†
†Dept. of Electrical and Electronic Engineering, Tokushima University
2-1 Minami-Josanjima, Tokushima 770–8506, Japan
Email: {yonex34, kentaago, uwate, nishio}@ee.tokushima-u.ac.jp
‡Murata Manufacting Co., Ltd. Email: kentaago@gmail.com

***Abstract*—Recently, it is said that it is important to analyze the damage of natural disasters and the process of spreading infectious diseases in a networked way to take countermeasures. The propagation of chaotic phenomena, in which complex waveforms are generated, are also focused, and chaotic propagation is studied by coupling many chaotic circuits to construct a network. In this study, the propagation path and the chaos generation rate of each circuit are studied by computer simulation. The correlation between the features of the network, such as the degree and the shortest path length, and the chaos propagation is also investigated.**

I. Introduction

In recent years, the damage caused by natural disasters and the process of spreading infectious diseases are studied focusing on the fact that they are network-like [1]. In natural disasters, damage spreads in the network-like way, with nodes as compound disasters and edges as cascading disasters, resulting in a wide variety of mixed damage. In infectious diseases, the structure of the network is that clusters are nodes, and edges extending from the nodes create new clusters. In order to prevent such damage from increasing, it is important to focus on the network and develop research on propagation.

It is also confirmed that propagation occurs in chaotic phenomena. The notion of propagation of chaos for large systems of interacting particles originates in statistical physics and has recently become a central notion in many areas of applied mathematics [2]. In addition, the propagation of chaos is studied by using chaotic circuits, which can be clearly described by physical laws, and coupling them with resistors to construct a network.

Then, what factors affect chaos propagation? For example, it could be the characteristics of the waveform, distance, path, network structure, and regulation. They can be considered as non-linearity, frequency, number of circuit to be passed, presence of hubs, clustering coefficient, and coupling strength in the networks coupled chaotic circuit.

The effects of frequency and coupling strength on phase difference and propagation speed were studied by generating chaos at one circuit in a ring coupled chaotic circuit [3], and the effects of hub presence and path distance were studied in multi-ring coupled chaotic circuits [4]. In addition, the effects of clustering coefficients and generation points were studied in complex networks [5]. As a result, it was found that the propagation occurred only between circuits set to the same frequency. In addition, it was found that propagation was easier when the path distance was small. Furthermore, when there were many hubs and the clustering coefficient was large, the propagation rate became small.

In this study, the propagation paths and the rate that chaos is generating are investigated in random and scale-free networks. Therefore, the order of propagation is shown by visualizing it in color and calculating the rate that chaos generating in each circuit for a certain time. Furthermore, the correlation between the results of propagation and network features is investigated.

II. System Model

The chaotic circuit is shown in Fig. 1. This circuit consists of a negative resistor, two inductors, a capacitor and dual-directional diodes. This circuit is considered as a node in the complex networks.

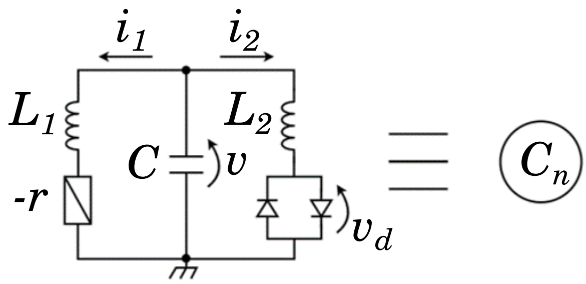


Fig. 1. Chaotic circuit.

In the proposed system, each circuit is connected to only adjacent circuits by the resistors. The normalized circuit equations of the system are given as follows:

$$\begin{cases} \dfrac{dx_n}{d\tau} = \alpha x_n + z_n \\ \dfrac{dy_n}{d\tau} = z_n - f(y_n) \\ \dfrac{dz_n}{d\tau} = -x_n - \beta y_n - \displaystyle\sum_{j \in S_n} \gamma(z_n - z_m) \\ \qquad\qquad (n, m = 1, 2, \cdots, N). \end{cases} \quad (1)$$

In Eq. (4), N is the number of coupled chaotic circuits and γ is the coupling strength.

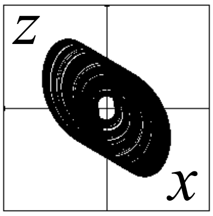
The characteristic equation for dual-directional diodes $f(y_n)$ is described as follows:

$$f(y_n) = \frac{1}{2}\left(\left|y_n + \frac{1}{\delta}\right| - \left|y_n - \frac{1}{\delta}\right|\right). \quad (2)$$

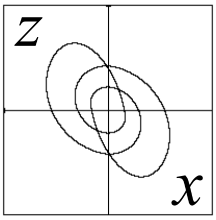
The parameters are described as follows:

$$\begin{cases} i_1 = \sqrt{\frac{C}{L_1}}Vx_n, \quad i_2 = \frac{\sqrt{L_1 C}}{L_2}Vy_n, \quad v = Vz_n \\ \alpha = r\sqrt{\frac{C}{L_1}}, \quad \beta = \frac{L_1}{L_2}, \quad \delta = r_d\frac{\sqrt{L_1 C}}{L_2}, \\ \gamma = \frac{1}{R}\sqrt{\frac{L_1}{C}}, \quad t = \sqrt{L_1 C_2}\tau. \end{cases} \quad (3)$$

In this chaotic circuit, we define α_c to generate the chaotic attractor (Fig. 2(a)) and α_p is defined to generate the three-periodic attractors (Fig. 2(b)). We calculate Eq. (4) using the fourth-order Runge-Kutta method with the step size h = 0.005 with the computer simulations. In this study, we set the parameters of the system as $\alpha_c = 0.460$, $\alpha_p = 0.413$, $\beta = 3.0$, $\gamma = 0.002$ and $\delta = 470.0$. After getting to the steady state that all the circuits generate three-periodic attractors, chaos propagation is simulated by generating chaos from one circuit.



(a) Chaos.



(b) Three-period.

Fig. 2. Attractors of chaotic circuit.

III. Complex Network Model

Figures 3 and 4 show the proposed different type network models. The characteristic of the complex network in Fig. 3 is the BA scale-free topology. The characteristic of the complex network in Fig. 4 is the ER random topology. In each model, each chaotic circuit is coupled by one resistor R. We use 25 coupled chaotic circuits and 46 resistors in each network model.

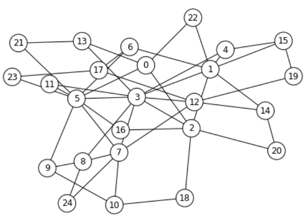


Fig. 3. BA scale-free network.

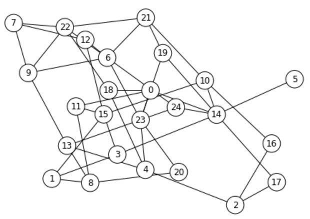


Fig. 4. ER random network.

Moreover, the feature quantities of the proposed each network are summarized in Table I. Topological structures in complex networks of N nodes and E edges can be evaluated by the typical three types structural metrics such as degree, clustering coefficient and path length. First, degree k is the number of edges which is connected on a node. Second, path length L shows the shortest path in the network between two nodes. Third, clustering coefficient C shows the number of actual links between neighbors of a node divided by the number of possible links between those neighbors.

TABLE I
FEATURE QUANTITIES OF PROPOSED EACH NETWORK.

Feature	BA scale-free	ER random
The number of nodes	25	25
The number of edges	46	46
Avg. degree	3.680	3.680
Avg. path length	2.363	2.453
Avg. clustering coefficient	0.165	0.114

Figure 5 shows degree distribution of each network. In this graph, the vertical axis denotes the probability of existence of degree and the horizontal axis denotes the value of degree.

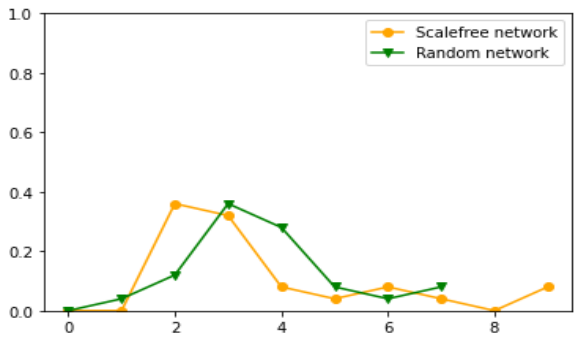


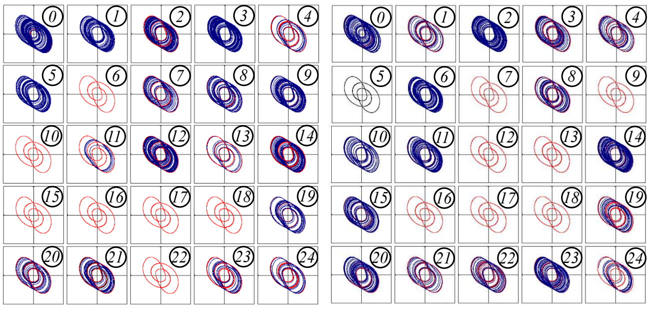
Fig. 5. Degree distribution of each network.

IV. Results

The order of chaos propagation and the rate that chaos is propagating in each circuit are investigated in the BA scale-free and the ER random. We explain the simulation method of the chaos propagation. First, the three-periodic attractors are generated with the same initial values of x_n, y_n and z_n for all the circuits. Then, the chaos is generated at node 0. In other words, the parameter $\alpha_p = 0.413$ of the circuit at node 0 is changed to $\alpha_c = 0.46$. The chaos propagation for 900,000 counts is simulated. At that time, the threshold width was set in the plane of $x_1 \geq 0, x_2 = 0$ to determine the three-periodic attractor and the chaotic attractor. When the three-periodic attractor drawn in black changes to the chaotic attractor, the chaotic attractor and waveform are drawn in blue. When it becomes the three-periodic attractor again, the three-periodic attractor is drawn in red.

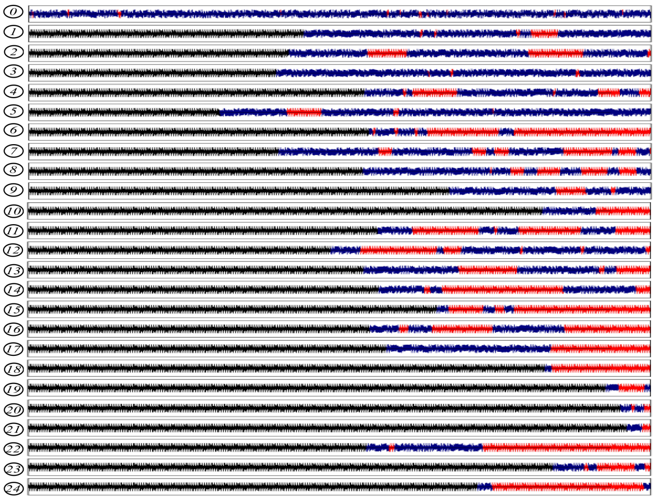
A. Attractors and wave forms

At about 900,000 counts after the generation of chaos, the attractors that the chaos propagated in the BA scale-free are shown in Fig. 6(a), and those in the ER random are shown in Fig. 6(b). For 900,000 counts after the generation of chaos, wave forms that the chaos propagated in the BA scale-free are shown in Fig. 7(a), and those in the ER random are shown in Fig. 7(b).

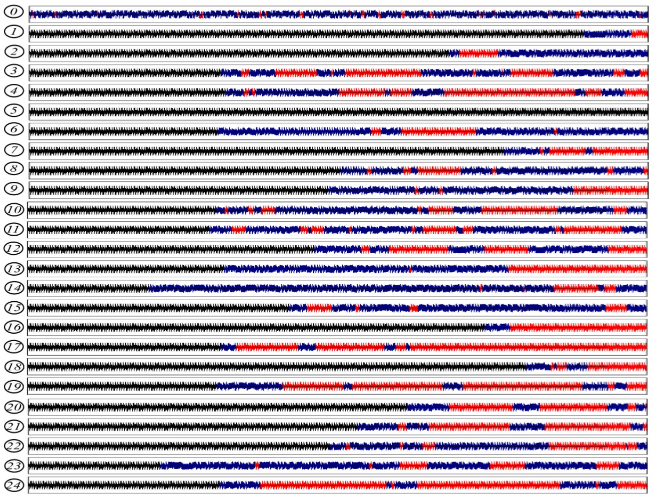


(a) BA scale-free. (b) ER random.

Fig. 6. Attractors.



(a) BA scale-free.



(b) ER random.

Fig. 7. Wave forms of x_n.

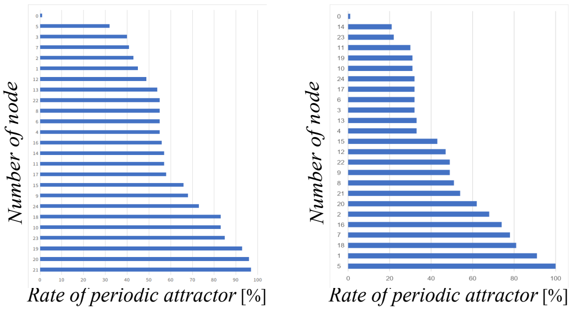
After the chaos propagated to most of the circuits, some of them returned to the three-periodic attractors. However, only node 5 in Fig. 6(b) remained the initial three-periodic attractor.

B. The Order of chaos propagation

The rate is calculated by counting the time when chaos is generating and the time when chaotic attractors become periodic attractor again. From those results, the rate of the three-periodic attractor in the initial state are shown in Fig. 8, and it indicates that the chaos propagates in the order of decreasing value.

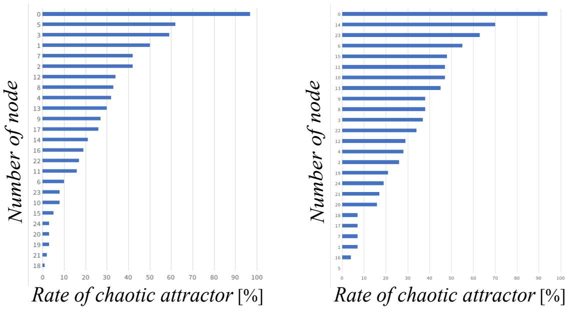
The start of propagation in the BA scale-free took longer than in the ER random. However, when the chaos propagated to the hub, it propagated quickly in large numbers in Fig. 8.

The rate of time that each circuit is generating chaos are shown in Fig. 9. As well as the previous studies [5], the rate that chaos is generating in BA scale-free is smaller than that in ER random.



(a) BA scale-free (b) ER random

Fig. 8. Rate of the three-periodic attractor in the initial state.



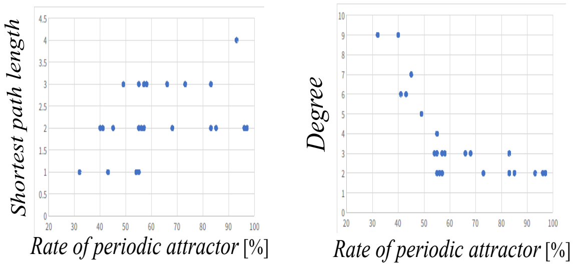
(a) BA scale-free (b) ER random

Fig. 9. Rate of the chaotic attractor.

C. Correlation

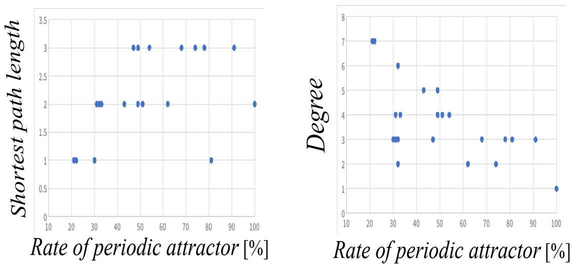
We investigate the correlation between the propagation of chaos and the features of the networks. First, the correlation between the rate of three-periodic attractor in initial state, that is mean, the order that the chaos is propagating, and the shortest path length from node 0 is investigated in Figs. 10-11(a). Secondly, the correlation between the order of propagation and the degree is investigated. The results are shown in Figs. 10-

11(b) and Table II. In addition, the correlation between the rate that chaos is generating and the features of those networks is investigated. The results are shown in Figs. 12-13 and Table III.



(a) Shortest path length (b) Degree

Fig. 10. Correlation between order chaos propagate and features of BA Scale-free.



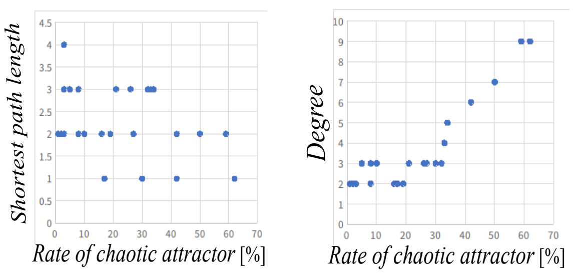
(a) Shortest path length (b) Degree

Fig. 11. Correlation between order chaos propagate and features of ER Random.

TABLE II
CORRELATION BETWEEN ORDER CHAOS PROPAGATE AND FEATURES OF EACH NETWORK.

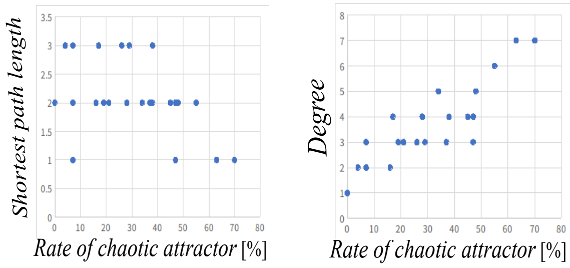
Feature	BA scale-free	ER random
Shortest path length	0.366	0.455
Degree	- 0.725	- 0.581

Investigating the correlation between the order of propagation and the features of the network, we found that the larger degree of the BA scale-free, the stronger the tendency to propagate. On the other hand, the ER random tended to propagate more easily when the shortest path length was shorter than that of the BA scale-free, although the correlation was weak.



(a) Shortest path length (b) Degree

Fig. 12. Correlation between rate chaos is generating and features of BA Scale-free.



(a) Shortest path length (b) Degree

Fig. 13. Correlation between rate chaos is generating and features of ER Random.

TABLE III
CORRELATION BETWEEN RATE CHAOS IS GENERATING AND FEATURES OF EACH NETWORK.

Feature	BA scale-free	ER random
Shortest path length	- 0.360	- 0.480
Degree	0.907	0.848

It was found that the larger the degree of both networks, the stronger the tendency for chaos to keep generating. On the other hand, the correlation with the shortest path length was similar to the result of the propagation order.

V. CONCLUSION

In this study, the order of chaos propagation and the rate that chaos is generating in complex networks were investigated. The correlations between these results and the features of complex networks were compared between ER random and BA scale-free. In the BA scale-free, the larger the degree, the faster the order of propagation, and the higher the rate that chaos is generating. In addition, the ER random was more correlated with the shortest path length than the BA scale-free. In the future, we will investigate whether the same trend can be obtained when the generation points are changed according to the degree. In addition, we will investigate the effect in a large scale network with 100 circuits. We are also considering to investigate the effect of reducing the parameter of the chaos of the generating point to the three-period attractor.

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