

Investigating Relationship between Synchronization Phenomena and Complex Network Properties by Using Chaotic Circuits

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1. Introduction

Synchronous phenomena like a frog chorus and a metronome can be observed in various parts of our lives. In this study, we focus on the synchronization of chaotic circuits. We investigate the synchronization rate from chaotic circuits connected in the network. In order to make the connection as similar as possible to a real network, three kinds of complex networks with different properties are used.

2. System Model

Figure 1 shows the chaotic circuit used in this study. The difference between the two chaotic circuits is whether dual-directional diodes or one-directional diode. The three network models are shown in Fig. 2. Network properties of three models are shown in Tab. 1.

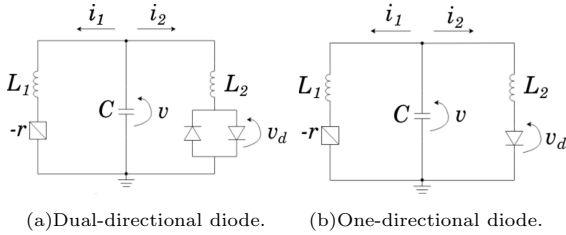


Figure 1: Chaotic circuits.

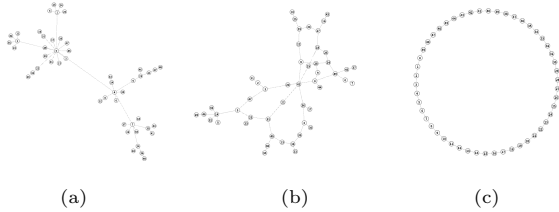


Figure 2: Complex network models ($N=50$). (a) BA scale-free. (b) ER random. (c) WS Regular.

Table 1: Network properties of three models (Node number N , Average node degree d_{avg} , Maximum node degree d_{max} and Average distance D_{avg}).

Network	BA scale-free	ER random	WS Regular
N	50	50	50
d_{avg}	1.96	2.2	2.0
d_{max}	14	5.0	2.0
D_{avg}	3.935	5.662	12.755

The normalized circuit equations are given as follows:

$$\begin{cases} \dot{x}_i = \alpha x_i + z_i \\ \dot{y}_i = z_i - f(y_i) \\ \dot{z}_i = -x_i - \beta y_i - \sum_{j=1}^{50} \gamma_{ij} (z_i - z_j). \end{cases} \quad (1)$$

where the function $f(y_i)$ is the equation for the diode and described as follows:

$$f(y_i) = \frac{1}{2} \left(\left| y_i + \frac{1}{\delta} \right| - \left| y_i - \frac{1}{\delta} \right| \right). \quad (2)$$

$$f(y_i) = \frac{1}{2} \left(y_i + \frac{1}{\delta} - \left| y_i - \frac{1}{\delta} \right| \right). \quad (3)$$

The Eq. (2) shows the characteristic equation of dual-directional diodes and the Eq. (3) shows one-directional diode.

3. Results

The parameters of the system are set to $\alpha = 0.412$, $\beta = 3.0$ and $\delta = 470$ for dual-directional diodes, $\delta = 9.0$ for one-directional diode. Here the initial values x , y , and z are generated in two different methods: regular and random and the ranges of the initial values are set to $[0.1 : 0.2]$. In this study, we focus on the relation between initial values differences, synchronization rates and network properties. Tables 2 and 3 show the simulation results in the three network models. Further we investigate the synchronization rates by changing the coupling strength. The BA model is excluded from the comparison because a sufficient number of measurements cannot be obtained due to its properties.

Table 2: Synchronization rates (dual-directional diodes).

Method	BA[%]	ER[%]	WS[%]
Regular	(35.5)	35.4	30.3
Random	(35.6)	34.1	29.8

Table 3: Synchronization rates (one-directional diode).

Method	BA[%]	ER[%]	WS[%]
Regular	(73.8)	90.1	72.9
Random	(79.9)	87.9	82.3

From Tabs. 2 and 3, the synchronization rates are high for one-directional diode. Further, it is confirmed that the ER model has relatively high synchronization rates and the rates are not obtained when coupling strength is increased in BA model.

4. Conclusion

In this study, it was considered that high synchronization rates could be obtained for network topology with small average distances. However, in the BA model of the circuit with high degree nodes, the high synchronization rates could not be obtained because the circuit would be heavily loaded and the circuit would not oscillate. There was no relationship between the way of the initial values were given and the synchronization rates.