Investigation of Clustering Phenomena in Coupled Chaotic Circuits Located in Four-Dimensional Space

Takumi Nara, Yoko Uwate and Yoshifumi Nishio
Dept. of Electrical and Electronic Engineering, Tokushima University
2-1 Minami-Josanjima, Tokushima 770-8506, Japan
Email: {nara, uwate, nishio}@ee.tokushima-u.ac.jp

***Abstract*—In this study, we investigate synchronization phenomena in the coupled chaotic circuits which are connected by resistor. In addition, we investigate the difference of synchronization phenomena by distance information of the coupled chaotic circuits and the difference of synchronization phenomena by increasing the number of coupled chaotic circuits. We confirm that the coupled chaotic circuits located in the near distance are synchronized at in-phase state, and the coupled chaotic circuits located in the long distance are not synchronized. Therefore, the clustering of coupled chaotic circuits are observed in two-dimensional, three-dimensional and four-dimensional space. In addition, the circuit simulation results and the k-means clustering results are compared and verified.**

I. Introduction

Synchronization phenomena are the most familiar phenomena that exist in nature and they have been studied in various fields. Synchronization phenomena can be observed everywhere in our life. For example, we can confirm flashing firefly lights, metronome, beating rhythm of the heart and so on. Especially, synchronization phenomena of oscillatory network are interesting. In addition, complex networks attract attention from various fields. The feature of networks is the degree distribution, the path length and the clustering coefficient. Therefore, we focus on the clustering phenomena in this study.

The clustering phenomena are to divide the set to be classified into subsets. Previously, many of the studies for clustering have been carried out for discrete time model, for example Coupled Map Lattices (CML) and Self Organization Map (SOM) and so on. Previously, many of the studies for clustering have been carried out for discrete time model [1]-[2]. However, analysis of using a continuous time model has not almost studied. Therefore, we focus on research on clustering phenomena using electronic circuits in the continuous time model. Therefore, we focus on research on clustering phenomena using electronic circuits in the continuous time model.

On the other hand, the coupled chaotic circuits that are electronic circuits can be observed various amusing phenomena. In recent years, many methods are studied to apply to clustering and synchronization phenomena observed in coupled chaotic circuits for natural sciences. At the same time, synchronization phenomena and clustering have been studied associated with the chaos phenomena [3]-[4].

In our previous study, we focus on the clustering phenomena in the network of coupled chaotic circuits. For this investigation, the coupling strength reflected the distance information when the chaotic circuits are located in two-dimensional, three-dimensional and four-dimensional space. First, we investigate from seven circuits in the two-dimensional space. From there, we increase and investigate the number of clusters. In the research so far, we were able to confirm the clustering of seven circuits in two-dimensional space and thirty circuits in three-dimensional space [5].

In this research, in addition to the research results so far, we study thirty circuits in four-dimensional space. Moreover, we investigate clustering by k-means clustering. The k-means clustering confirms the demonstrability of research performed on circuits by performing simulations with the same location information as circuit simulations. Furthermore, the accuracy is verified by comparing it with the results of circuit simulation.

II. Circuit Model

Figure 1 shows the circuit model which is called Shinriki-Mori circuit.

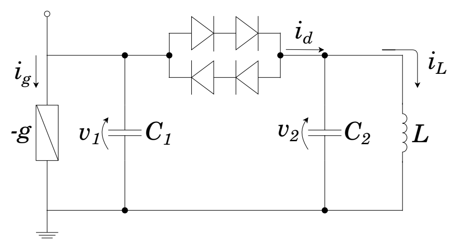


Fig. 1. Circuit model.

By changing the variables and parameters such that

$$i_L = \sqrt{\frac{C_2}{L}}Vx, \quad v_1 = Vy, \quad v_2 = Vz$$

$$\alpha = \frac{C_2}{C_1}, \quad \beta = G_d\sqrt{\frac{L}{C_2}}, \quad \gamma = g\sqrt{\frac{L}{C_2}}, \quad t = \sqrt{LC_2}\tau$$

The normalized equation of chaotic circuit is given as follows:

$$\begin{cases} \dfrac{dx}{d\tau} = z \\ \dfrac{dy}{d\tau} = \alpha(\gamma y - \beta f) \\ \dfrac{dz}{d\tau} = \beta f - x. \end{cases} \quad (1)$$

The nonlinear functions f corresponds to the characteristics of the nonlinear resistor consisting of the diodes and described as follows:

$$f = \begin{cases} y - z - 1 & (y - z > 1) \\ 0 & (|y - z| \leq 1) \\ y - z + 1 & (y - z < -1). \end{cases} \quad (2)$$

For the computer simulation, we set the parameters as $\alpha = 0.50$, $\beta = 20.00$ and $\gamma = 0.50$.

III. Simulation Results

A. Network of seven chaotic circuits in two-dimensional space

First, we investigate the synchronization phenomena and clustering phenomena when seven chaotic circuits are coupled in two-dimensional space. The location of seven chaotic circuits is shown in Fig. 2 and Table I.

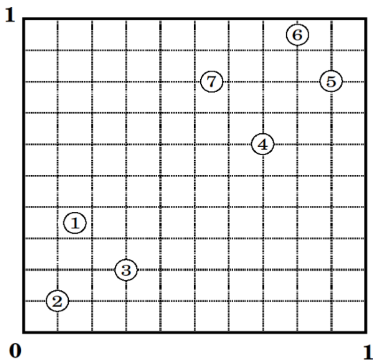


Fig. 2. Location of seven chaotic circuits in two-dimensional space.

TABLE I
THE LOCATION OF CHAOTIC CIRCUITS IN THE TWO-DIMENSIONAL SPACE.

No.	x	y
1	0.15	0.35
2	0.10	0.10
3	0.30	0.20
4	0.70	0.60
5	0.90	0.80
6	0.80	0.95
7	0.55	0.80

All circuits connect each other by resistors. Figure 3 shows coupling method of the first chaotic circuit as an example.

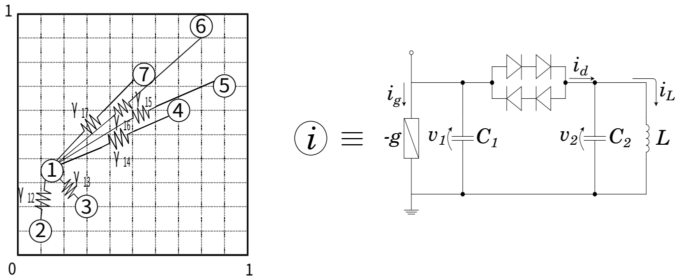


Fig. 3. Coupling between the first chaotic circuit and others.

We consider the coupled chaotic circuits:

$$\begin{cases} \dfrac{dx_i}{d\tau} = z_i \\ \dfrac{dy_i}{d\tau} = \alpha(\gamma y_i - \beta f - \displaystyle\sum_{i,j=1}^{N} r_{i,j}(y_i - y_j)) \\ \dfrac{dz_i}{d\tau} = \beta f - x_i. \end{cases} \quad (3)$$

The nonlinear functions f corresponds to the i-v characteristics of the nonlinear resistors consisting of the diodes and are given as follows:

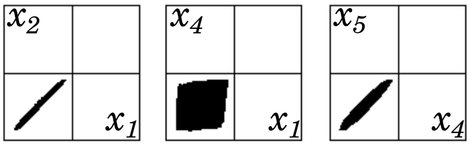
$$f = \begin{cases} y_i - z_i - 1 & (y_i - z_i > 1) \\ 0 & (|y_i - z_i| \leq 1) \\ y_i - z_i + 1 & (y_i - z_i < -1). \end{cases} \quad (4)$$

where, i in the equation represents the circuit itself, and j is the coupling with other circuits. The parameter r represents the coupling strength between the circuits. In this simulation, we set the coupling parameter value $r_{i,j}$ to correspond the distance between the circuits by the following equation:

$$r_{i,j} = \frac{q}{(d_{i,j})^2}. \quad (5)$$

$d_{i,j}$ represents the Euclidean distance between the $i-th$ and the $j-th$ circuits. Further, the parameter q is the weight parameter that determines the coupling strengths. In this case, we set parameter q = 0.01.

Figure 4 shows the computer simulation results obtained from the seven chaotic circuits located as shown in Fig. 2. From these results, we confirm that the first, second and third chaotic circuits are synchronized at in-phase state, and also the fourth, fifth, sixth and seventh chaotic circuits are synchronized at in-phase state. However, the first and the fourth chaotic circuits are not synchronized. From these results, the circuits can form two clusters defined by chaotic synchronization as shown in Fig. 5.



(a)1-2 (b)1-4 (c)4-5

Fig. 4. Phase difference between seven circuits in two-dimensional space.

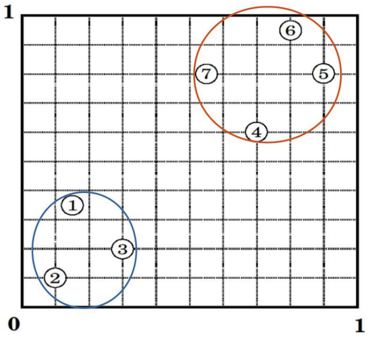


Fig. 5. The clustering result of seven chaotic circuits.

Next, we studied clustering by the k-means clustering. The result is shown in Fig. 6. From Fig. 6, the result is similar to the simulation results performed in the circuits. Therefore, we can be proved that clustering can be reproduced even in the circuit simulation.

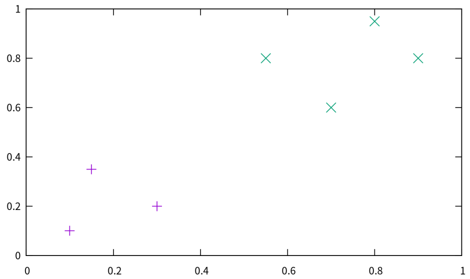


Fig. 6. The result of k-means clustering.

B. Network of thirty chaotic circuits in three-dimensional space

Next, we investigate the case of three-dimensional networks. Thirty chaotic circuits are located in three-dimensional, including the location information. The location of thirty chaotic circuits is shown in Fig. 7 and Table II. In this case, we put the parameter $q = 0.004724$.

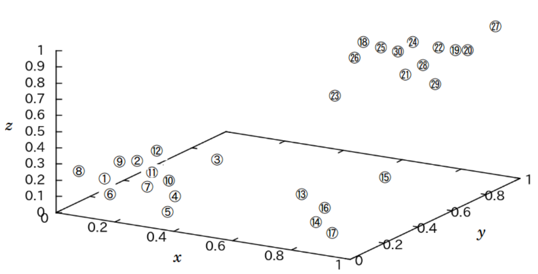
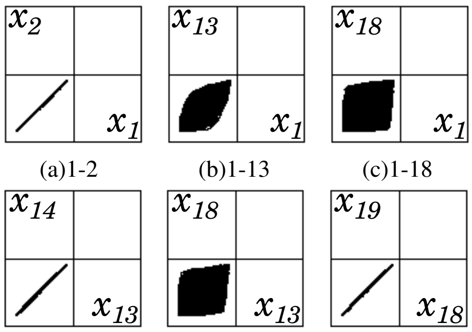


Fig. 7. Location of thirty chaotic circuits in three-dimensional space.

TABLE II
THE LOCATION OF CHAOTIC CIRCUITS IN THE THREE-DIMENSIONAL SPACE.

No.	x	y	z	No.	x	y	z
1	0.15	0.05	0.15	16	0.80	0.20	0.15
2	0.20	0.25	0.30	17	0.85	0.15	0.05
3	0.35	0.35	0.25	18	0.70	0.60	0.95
4	0.25	0.25	0.05	19	0.90	0.80	0.85
5	0.30	0.15	0.05	20	0.80	0.95	0.75
6	0.05	0.20	0.10	21	0.75	0.85	0.70
7	0.15	0.30	0.15	22	0.85	0.80	0.85
8	0.05	0.05	0.25	23	0.60	0.60	0.60
9	0.20	0.05	0.35	24	0.80	0.65	0.90
10	0.25	0.10	0.20	25	0.65	0.80	0.80
11	0.35	0.05	0.25	26	0.65	0.65	0.85
12	0.25	0.05	0.35	27	0.95	0.95	0.95
13	0.75	0.15	0.25	28	0.90	0.65	0.75
14	0.80	0.25	0.10	29	0.70	0.85	0.65
15	0.95	0.30	0.35	30	0.75	0.80	0.85

Figure 8 shows the computer simulation results obtained from the thirty chaotic circuits located as shown in Fig. 7. From these results, we confirm that the first and the second circuits are synchronized at in-phase state. However, the first chaotic circuit and the thirteenth chaotic circuit are not synchronized. Also the first chaotic circuit and the eighteenth chaotic circuit are not synchronized. Similarly, between the thirteenth and the fourteenth chaotic circuit are synchronized, between the eighteenth and the nineteenth chaotic circuit are synchronized. However, between the thirteenth and the eighteenth chaotic circuit are not synchronized. From these results, the circuits can form three clusters defined by chaotic synchronization as shown in Fig. 8. Furthermore, the simulation by the k-means clustering is performed as in the case of two-dimensional research. Figure 10 shows the result. From Fig. 10, we were able to obtain the same result as the two-dimensional research.



(a)1-2 (b)1-13 (c)1-18

(d)13-14 (e)13-18 (f)18-19

Fig. 8. Phase difference between thirty circuits in three-dimensional space.

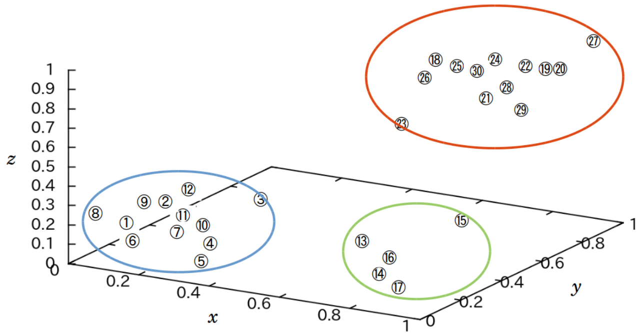


Fig. 9. The clustering result of thirty chaotic circuits.

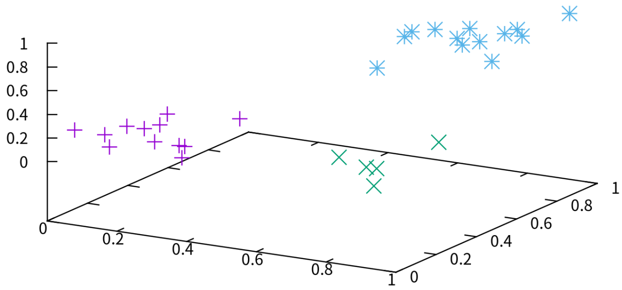


Fig. 10. The result of k-means clustering.

C. *Network of thirty chaotic circuits in four-dimensional space*

Next, we investigate the case of four-dimensional networks. Thirty chaotic circuits are located in four-dimensional, including the location information. The location of thirty chaotic circuits is shown in Table III. In this case, we put the parameter $q = 0.005325$.

TABLE III
THE LOCATION OF CHAOTIC CIRCUITS IN THE FOUR-DIMENSIONAL SPACE.

No.	a_1	a_2	a_3	a_4	No.	a_1	a_2	a_3	a_4
1	0.15	0.05	0.15	0.25	16	0.80	0.20	0.15	0.85
2	0.20	0.25	0.30	0.10	17	0.85	0.15	0.05	0.75
3	0.35	0.35	0.25	0.05	18	0.70	0.30	0.20	0.70
4	0.25	0.25	0.05	0.15	19	0.90	0.80	0.85	0.05
5	0.30	0.15	0.05	0.20	20	0.80	0.95	0.75	0.10
6	0.05	0.20	0.10	0.30	21	0.75	0.85	0.70	0.20
7	0.15	0.30	0.15	0.80	22	0.85	0.80	0.85	0.25
8	0.05	0.05	0.25	0.90	23	0.70	0.60	0.95	0.35
9	0.20	0.05	0.35	0.70	24	0.80	0.65	0.90	0.15
10	0.25	0.10	0.20	0.75	25	0.65	0.80	0.80	0.75
11	0.35	0.05	0.25	0.80	26	0.65	0.65	0.85	0.70
12	0.25	0.05	0.35	0.85	27	0.95	0.95	0.95	0.65
13	0.75	0.15	0.25	0.20	28	0.90	0.65	0.75	0.90
14	0.80	0.25	0.10	0.15	29	0.70	0.85	0.65	0.85
15	0.95	0.30	0.35	0.05	30	0.75	0.80	0.85	0.80

Figure 11 shows the computer simulation results. From these results, we confirm that the first and the second circuits are synchronized at in-phase state. However, the first chaotic circuit and the seventh, the thirteenth, the sixteenth, the nineteenth and the twenty-fifth chaotic circuit are not synchronized. Similarly, between the seventh and the eighth, the thirteenth and the fourteenth, the sixteenth and the seventeenth, the nineteenth and the twentieth, the twenty-fifth and the twenty-sixth chaotic circuit are synchronized. However, the seventh chaotic circuit and the thirteenth, the sixteenth, the nineteenth and the twenty-fifth chaotic circuit, the thirteenth chaotic circuit and the sixteenth, the nineteenth and the twenty-fifth chaotic circuit, the sixteenth chaotic circuit and the nineteenth and the twenty-fifth chaotic circuit, the nineteenth chaotic circuit and the twenty-fifth chaotic circuit are not synchronized. From these results, the circuits can form six clusters defined by chaotic synchronization. After that, k-means clustering is performed as in the three-dimensional research. We were able to obtain the same result as the circuit simulation as before.

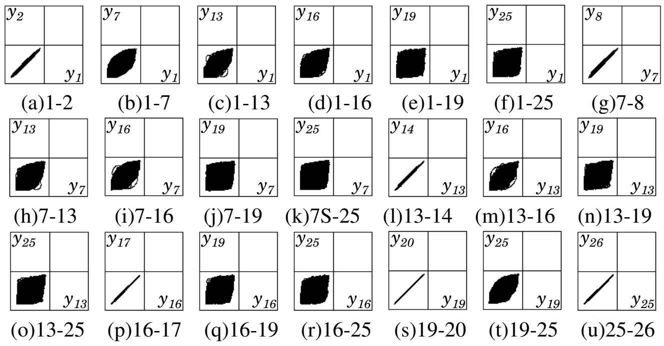


Fig. 11. Phase difference between thirty circuits in four-dimensional space.

IV. Conclusion

In this study, we investigated synchronization phenomena when the chaotic circuits are located in two-dimensional, three-dimensional and four-dimensional space. Synchronization phenomena were seen between circuits at near distance, and synchronization phenomena could not be seen between circuits at far distance. With these results, it was confirmed that the chaotic circuits were different from synchronization phenomena by distance information and the clustering phenomena were observed. From the results of the k-means clustering, we were able to verify that the circuit synchronization phenomena can be used for clustering.

In the future works, we would like to increase the number of chaotic circuits and the cluster. Moreover, we consider that we would like to change of the dimensional space. We also would like to investigate changes in the clusters by changing the value of the coupling strength q.

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