

Simulation of Infectious Number of COVID-19 in JAPAN by Using SIR Model with Echo State Networks

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Abstract—In this study, we propose a model to simulate infectious number of COVID-19 in Japan by using a mathematical model that is called SIR model. We use time variables for SIR model, and Echo State Networks (ESN) is applied to simulate parameters of SIR model. Then, we use the simulated parameters, and infectious number in Japan is estimated.

I. INTRODUCTION

COVID-19 is confirmed for the first time in the city of Wuhan on December 2019. As everyone knows, COVID-19 spread all over the world, and still now on confirmed cases are increasing day by day. According to Jonhs Hopkins University's report, more than 50,000,000 infected cases and 1,200,000 deaths had been confirmed globally until November 11th 2020 [1]. Then, many cities and countries are locked down to prevent spreading COVID-19, and some of cities are enforced lock down before pandemic or outbreak situation. As a result, they could prevent pandemic. Therefore, it is important to predict infectious numbers, and to know how the infectious number would go on.

Mathematical models for infectious diseases such as SIR model and SEIR model have been used for the simulation of infectious diseases. SIR model and SEIR model are used for Severe Acute Respiratory Syndrome and seasonal influenza as well [2],[3].

In addition, Echo State Networks (ESN) is one kind of neural networks, and ESN is used for time-series data computing.

In this study, we simulate infectious number of COVID-19 in Japan by using SIR model. Then, ESN is used to train and test the parameters of SIR model, and we aim at simulating the infectious graph that has more realistic waves.

II. SIR MODEL

SIR model is a popular mathematical model for infectious diseases. SIR is the initial letters of susceptible, infectious and removed. Susceptible indicates the statement of the people who do not have antibodies of that infection. In this study, as to COVID-19, we regard all of the people at the first point as susceptible. Infectious means the statement of the people who have infectious capacity. Removed means the statement of the people who recover from that infection and also people

who pass away. In SIR model, susceptible people move to the statement of infectious with the infectious rate α , and the infectious people move to the statement of removed with the removed rate γ . Figure 1 shows the statement structure of SIR model.

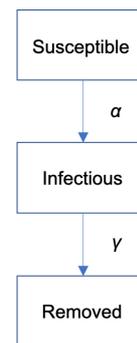


Fig. 1. The statement structure of SIR model.

This SIR model can be expressed by the following differential equations. $S(t)$, $I(t)$ and $R(t)$ mean the population of susceptible, the population of infectious and the population of removed at time t , respectively. Equation (1) shows the differential equations of SIR model.

$$\begin{cases} \frac{dS}{dt} = -\alpha S(t)I(t) \\ \frac{dI}{dt} = \alpha S(t)I(t) - \gamma I(t) \\ \frac{dR}{dt} = \gamma I(t) \end{cases} \quad (1)$$

In SIR model, all of the population are divided in these three statements which are susceptible, infectious and removed. Then, whole population N is the addition of the three statements as shown in Eq. (2), and we regard N as constant.

$$N = S(t) + I(t) + R(t) \quad (2)$$

Then, Eq. (1) is solved by the Runge-Kutta method. Figure 2 shows a typical graph of SIR model.

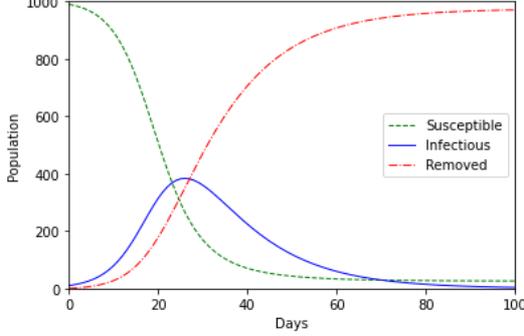


Fig. 2. A graph of SIR model.

As shown in Fig. 2, the infectious graph of SIR model has only one peak. SIR model has the advantage that is quick simulation and the simple structure. However, SIR model has the disadvantage that SIR model can not express multiple waves with constant parameters. Therefore, we aim at expressing multiple peaks in infectious simulation graph by applying ESN which is one kind of neural networks and which is introduced in the following chapter.

III. ECHO STATE NETWORKS

ESN is one kind of reservoir computing, and it can handle time-series data [4]. ESN has the feature of quick learning. Therefore, the advantage of SIR model would not be lose by using this network. ESN has reservoir layer between input and output. Figure 3 shows the structure of ESN.

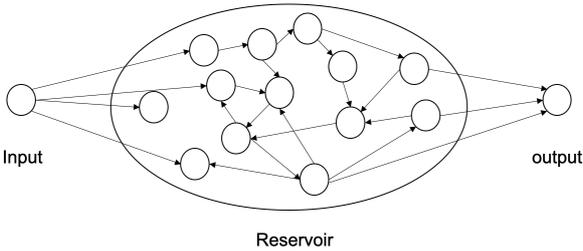


Fig. 3. The structure of Echo State Networks.

In the reservoir layer there is a lot of reservoir nodes, and the reservoir nodes are connected randomly. The input weights and reservoir weights are not updated, only the output weights are updated.

$x(n)$ is a vector of reservoir neuron activation states at time step n , and n is the discrete time. Tanh function is used as the activation function as shown in Eq. (3). $u(n)$ is the input data, a is the leaking rate and W_{in} is the input weight.

The update equations are shown in Eq. (3).

$$\begin{cases} \tilde{x}(n) = \tanh(W_{in}[1; u(n)] + Wx(n-1)) \\ x(n) = (1-a)x(n-1) + a\tilde{x}(n) \end{cases} \quad (3)$$

Root Mean Squared Percentage Error (MSE) is used for validation. N , $Y_{simulated}$ and Y^t means the number of the data, the simulation value and the target value, respectively.

$$MSE = \frac{1}{N} \sum_{i=1}^N (Y^t - Y_{simulated})^2 \quad (4)$$

W_{out} is the output weights, reg is the regularization coefficient, and I is the identity matrix in Eq. (5). X indicates $[1; u(n); x(n)]$.

$$W_{out} = Y^t X^T (X X^T + reg I)^{-1} \quad (5)$$

In ESN, W_{out} is learned and updated to minimize MSE . In this study, ESN is used for simulating of the parameters $\alpha(t)$ and $\gamma(t)$.

IV. DATA

We use open data from Ministry of Health, Labour and Welfare of Japanese government [5]. From the open data source, positive cases, recovered cases and death toll in Japan are obtained.

Then, $S(t)$, $I(t)$ and $R(t)$ are calculated by the following equations. $P(t)$, $R_e(t)$ and $D(t)$ means the reported positive cases, reported recovered cases and reported death toll at time t .

$$\begin{cases} S(t) = N - I(t) - R(t) \\ I(t) = \sum_{i=1}^t P(i) - \sum_{i=1}^t R(i) \\ R(t) = \sum_{i=1}^t R_e(i) - \sum_{i=1}^t D(i) \end{cases} \quad (6)$$

In this study, N is 126,000,000 that is the whole population of Japan.

V. PROPOSED METHOD

First, the parameters $\alpha(t)$ and $\gamma(t)$ are calculated from SIR model differential equations by using difference equations method.

$$\begin{cases} \alpha(t) = \frac{S(t) - S(t+1)}{S(t)I(t)} \\ \gamma(t) = \frac{R(t+1) - R(t)}{I(t)} \end{cases} \quad (7)$$

Second, the parameters $\alpha(t)$ and $\gamma(t)$ are learned and tested by ESN. It is iterated 30 times with random number for initial weight, and average output is used.

Third, $I(t)$ is calculated by using simulated parameters $\alpha(t)$ and $\gamma(t)$.

$$\begin{cases} S(t+1) = S(t) - \alpha(t)S(t)I(t) \\ R(t+1) = \gamma(t)I(t) + R(t) \\ I(t+1) = N - S(t+1) - R(t+1) \end{cases} \quad (8)$$

As to real data, we use first 100 data for training and the next 100 data for testing.

VI. SIMULATION RESULTS

Figure 4 shows the real data of $\alpha(t)$ which is calculated by Eq. (7).

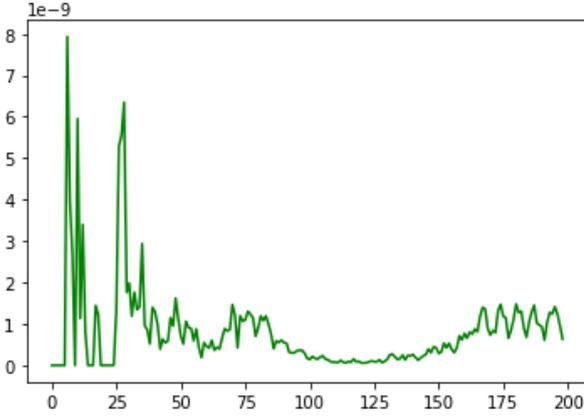


Fig. 4. The calculated real data of $\alpha(t)$.

Figure 5 shows the test result of $\alpha(t)$.

The following parameters are used for ESN. The size of reservoir is 50, the leaking rate is 0.15, the weight is 1.5 and the regularization coefficient is 0.5. As a result, MSE of $\alpha(t)$ is $2.17192e-19$.

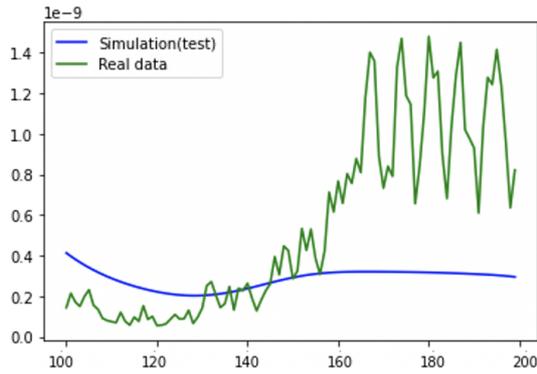


Fig. 5. The test result of $\alpha(t)$.

The test graph seems not to be following with the real data graph enough. Between 100th day to 140th day, the test graph seems to be following with the real data, but after 140th day the test graph shows different waveform and values with the real data.

Figure 6 shows the real data of $\gamma(t)$ which is calculated by Eq. (7).

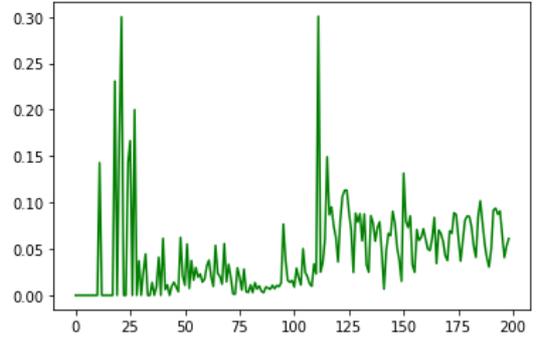


Fig. 6. The calculated real data of $\gamma(t)$.

The following parameters are used for ESN. The size of reservoir is 50, the leaking rate is 0.8, the weight is 1.1 and the regularization coefficient is $1.0e-2$. As a result, MSE of $\gamma(t)$ is 0.0030813.

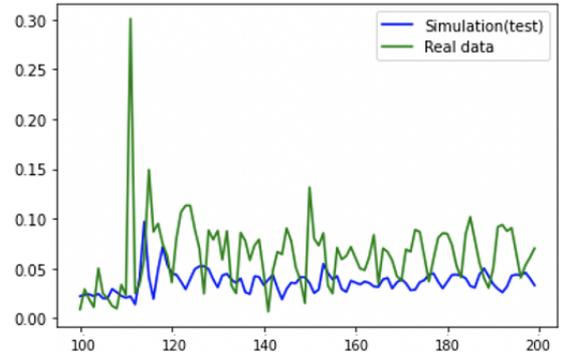


Fig. 7. The test result of $\gamma(t)$.

The test graph seems to be following with the real data graph. The serrate waveform which is shown in the real graph is expressed in the test graph. In addition, though the values of test graph have differences with the real data, it seems to be big differences compared with the result of $\alpha(t)$.

Figure 8 shows the simulation result of infectious number.

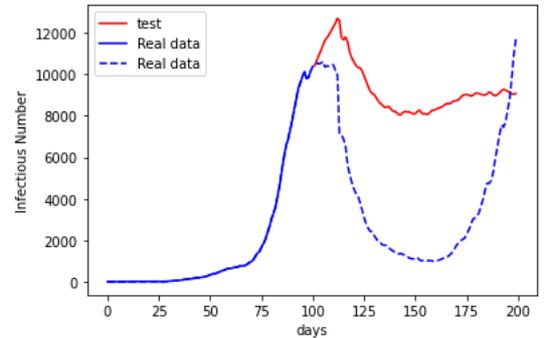


Fig. 8. The simulation result of infectious number for 300th days.

The waveform is not like a curb, it is serrate waveform. These features are not shown in the typical infectious graph of SIR model. Moreover, the valley of the simulation test graph is the same with the valley of real data. However the simulation test result is not following with the real data, especially as to the infectious number.

VII. CONCLUSIONS

In this study, infectious number was simulated with time variables $\alpha(t)$ and $\gamma(t)$ by using SIR model method and ESN to simulate the parameters $\alpha(t)$ and $\gamma(t)$. Then, we could express serrate waveform in the test part of infectious number simulation.

For future works, we need to improve the test accuracy of the parameters $\alpha(t)$ and $\gamma(t)$. Especially, we need to improve $\alpha(t)$ simulation. In this study, though we used 100 data for training and other 100 data for testing, these data size might be too small data size for ESN. However, to increase the data size is not good method. We will keep using small data size to predict for unknown infectious diseases besides COVID-19. To improve $\alpha(t)$ simulation, we will try to use other neural networks such as Convolutional Neural Networks and other Recurrent Neural Networks as well.

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