# Synchronization Phenomena in Coupled Two-Degrees-of-Freedom Chaotic Circuits by a Resistor

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# 1. Introduction

Asynchronous simultaneous oscillation is one of the typical non-linear phenomena and it is considered to exist in various high-dimensional systems in nature. Highdimensional chaotic circuits are actively studied, and a two-degrees-of-freedom chaotic circuit has been proposed [1]. We investigate synchronization phenomena when two its chaotic circuits with asynchronous simultaneous oscillation are connected by a resistor.

#### 2. System Model

The circuit model is shown in Fig. 1. Two two-degreesof-freedom chaotic circuits are coupled by a resistor R.



Figure 1: Circuit model.

The normalized circuit equations are described as follows:

$$\begin{aligned} \dot{x_{11}} &= -(y_{11} + z_{11}) + \alpha(x_{11} + x_{12}) - \delta(x_{11} + x_{12} - x_{21} - x_{22}) \\ \dot{y_{11}} &= x_{11} \\ \dot{z_{11}} &= \beta_{11}(x_{11} - f(z_{11})) \\ \dot{x_{12}} &= -\gamma_1(y_{12} + z_{12}) + \alpha\gamma_1(x_{11} + x_{12}) - \gamma_1\delta(x_{11} + x_{12} - x_{21} - x_{22}) \\ \dot{y_{12}} &= \beta_{12}x_{12} \\ \dot{z_{12}} &= \beta_{13}(x_{11} - f(z_{12})) \\ \dot{x_{21}} &= -\gamma_{21}(y_{21} + z_{21}) + \alpha\gamma_{21}(x_{21} + x_{22}) + \gamma_{21}\delta(x_{11} + x_{12} - x_{21} - x_{22}) \\ \dot{y_{21}} &= \beta_{21}x_{21} \\ \dot{z_{21}} &= \beta_{22}(x_{21} - f(z_{21})) \\ \dot{x_{22}} &= -\gamma_{22}(y_{22} + z_{22}) + \alpha\gamma_{22}(x_{21} + x_{22}) + \gamma_{22}\delta(x_{11} + x_{12} - x_{21} - x_{22}) \\ \dot{y_{22}} &= \beta_{23}x_{22} \\ \dot{z_{22}} &= \beta_{24}(x_{21} - f(z_{22})). \end{aligned}$$

The characteristic equation for the diode is described as follows:

$$f(x) = \frac{1}{2\varepsilon} (x + \varepsilon - |x - \varepsilon|).$$
(2)

### 3. Results

In this study, three natural frequency cases are considered. The frequencies of  $x_{11}$ ,  $x_{21}$ ,  $x_{12}$ ,  $x_{22}$  are corresponding to  $\omega_1$ ,  $\omega_2$ ,  $\omega_3$ ,  $\omega_4$ , respectively. Figure 2 shows the computer simulation results.





Figure 2: Attractors and synchronization phenomena.

In the case Fig. 2(a), in-phase synchronization phenomena are confirmed between the upper subcircuits and between the lower subcircuits. The circuit parameters are chosen as  $\alpha = 0.2$ ,  $\beta_{11} = 6.0$ ,  $\beta_{12} = 3.0$ ,  $\beta_{13} = 18.0$ ,  $\beta_{21} = 1.0$ ,  $\beta_{22} = 6.0$ ,  $\beta_{23} = 3.0$ ,  $\beta_{24} = 18.0$ ,  $\gamma_1 = 3.0$ ,  $\gamma_{21} = 1.0$ ,  $\gamma_{22} = 3.0$ ,  $\delta_{=}0.1$ , and  $\varepsilon = 0.01$ . In the case Fig. 2(b), inphase synchronization phenomena are confirmed only between the upper subcircuits. The circuit parameters are changed from case Fig. 2(a) to  $\beta_{12} = 2.0$ ,  $\beta_{13} = 12.0$ ,  $\gamma_1 = 2.0$ . In the case Fig. 2(c), although the subcircuits closest to the connection is asynchronous, it is confirmed that the lower subcircuits far from the connection are in-phase synchronized. The circuit parameters are changed from case Fig. 2(a) to  $\beta_{21} = 2.0$ ,  $\beta_{22} = 12.0$ ,  $\gamma_{21} = 2.0$ .

### (1) <u>4. Conclusion</u>

In this study, we confirmed that directly connecting two previously separated circuits via a resistor induced synchronization between the two underlining subcircuits farthest from the newly created connection, while the two closest subcircuits stayed asynchronous. As our future works, we will investigate synchronization phenomena in the circuit experiment.

### <u>References</u>

 Katunori SUZUKI, Yoshifumi NISHIO, and Shinsaku MORI, "Twin Chaos - Simultaneous Oscillation of Chaos -", The Institute of Electronics, Information and Communication Engineers (IEICE), vol. j79-A, No. 3, pp. 813-819, Mar. 1996.