Synchronization Phenomena in Chaotic Complex Networks with Coupling Strength and Factor of Synchronization

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***Abstract*—We investigate the influence of directly coupling and indirectly coupling of chaotic circuits for synchronization state in the network. The node of the network is expressed by a chaotic circuit. This complex network is designed that node has two or more couplings except for one node with scale free property. The coupling strength is determined depending on the number of couplings between the nodes. In this investigation, we find that form of network is important.**

I. Introduction

In our living life, complex network can be seen in various fields such as airport network, computer network and neurons in the brain. Furthermore, synchronization phenomena observed in complex network have been investigated by many researchers [1], [2]. Especially, synchronization phenomena of coupled oscillatory systems are very interesting. There are various investigations of synchronization using chaotic circuits. However, few studies focus on the difference in coupling strength [3] - [5].

In this study, we focus on the path and coupling strength of coupled chaotic circuit network. Path is indirectly connection formed by several circuits. Indirectly coupling refers to a connection through multiple circuits. We design a network that each node has different coupling strength depending the number of degree and the network has scale free property. Degree refers to the difference in the number of couplings between circuits that are coupled to each other. In order to make the network which the total coupling strength of node in the network has nearly uniform, we apply the following rule to set the coupling strength. When the difference of number of degree between two nodes is large, the coupling strength between two nodes is set to small value.

We use a chaotic circuit called "Shinriki-Mori Circuit" as a node. In this simulation, we investigate two computer simulation using this circuit. First, we compare all the synchronization of the connections of the hub. Hub is a node with many links in the network. All couplings were classified according to coupling strength. We make graphs for each value of coupling strength and compare.

Second, we investigate synchronization on indirectly coupling. We focus on one coupling. The coupling strength was changed of indirectly coupling. We investigate what kind of synchronization between the circuits.

II. Circuit Model

Figure 1 shows the chaotic circuit which is three dimensional autonomous circuit proposed by Shinriki et al. This circuit consists of one negative resister, two capacitors, one inductor and dual-directional three diodes. This circuit proposed by Shinriki *et al* [6], [7]. This circuit equation is shown in Eq. (1).

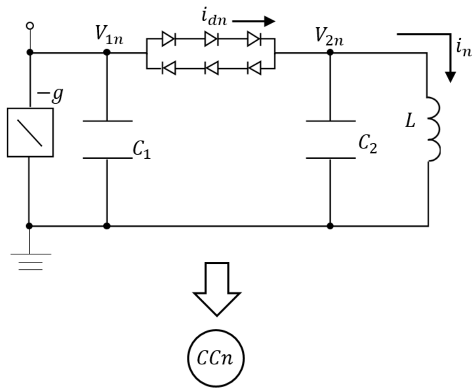

Fig. 1: Circuit model.

$$\begin{cases} L\dfrac{di_1}{dt} = V_{2n}, \\ C_1\dfrac{dV_{1n}}{dt} = gV_{1n} - i_{dn}, \\ C_2\dfrac{dV_{2n}}{dt} = i_{dn} - i_n. \end{cases} \quad (1)$$

The characteristic of nonlinear resistance which consists of dual three diodes is following Eq. (2).

$$i_{dn} = \begin{cases} Gd(V_{1n} - V_{2n} - V) & (V_{1n} - V_{2n} > V), \\ 0, & (|V_{1n} - V_{2n}| < V), \\ Gd(V_{1n} - V_{2n} + V), & (V_{1n} - V_{2n} < -V). \end{cases} \quad (2)$$

By changing the variables and parameters Eq. (3),

$$\begin{cases} i_n = \sqrt{\frac{C_2}{L}} V x_n, \quad V_{1n} = V y_n, \quad V_{2n} = V z_n \\ t = \sqrt{LC_2}\tau, \quad \alpha = \frac{C_2}{C_1}, \\ \beta = \sqrt{\frac{L}{C_2}} Gd, \quad \gamma = \sqrt{\frac{L}{C_2}} g, \quad \delta = \frac{1}{R}\sqrt{\frac{L}{C_2}}. \end{cases} \quad (3)$$

The normalized equations of this circuits are given as follows:

$$\begin{cases} \frac{dx}{d\tau} = z_n, \\ \frac{dy}{d\tau} = \alpha\gamma y_n - \alpha f(y_n - z_n), \\ \frac{dz}{d\tau} = f(y_n - z_n) - x_n. \end{cases} \quad (4)$$

where $f(y_n - z_n)$ is described as follows :

$$f(y_n - z_n) = \begin{cases} \beta(y_n - z_n - 1) & (y_n - z_n > 1), \\ 0, & (|y_n - z_n| < 1), \\ \beta(y_n - z_n + 1), & (y_n - z_n < -1). \end{cases} \quad (5)$$

III. System model

This complex network is designed that each node has two or more couplings with Scale-free property. Only three nodes are set to the hubs (CC18, CC19, CC20). All coupling strength values are fixed with a certain rule. This rule is based on the difference of the node degrees between nodes. The value of coupling strength is determined by the number of degree of the node. Namely, the coupling strength is set to the certain value by calculating the difference of number of degree between the nodes and add 1.0 for every couplings. Eq. (6) is equation of coupling strength.

$$\begin{cases} Coupling\ strength = \frac{1.0}{1.0 + u} \end{cases} \quad (6)$$

In this equation, u is difference of node degrees.

Initial value of all circuits have been changed. We have to use initial value sensitivity.

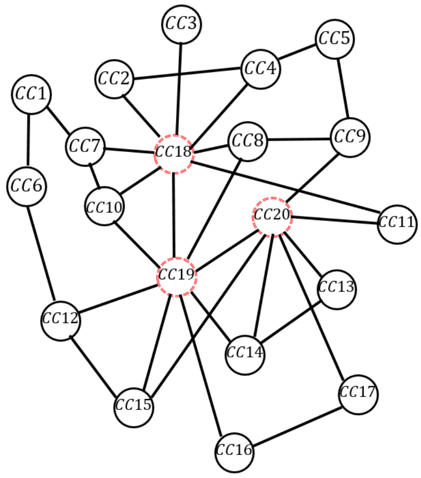

Fig. 2: Original network model.

TABLE I: Coupling strength.

Difference of Digree	7	6	5	4
Coupling strength	0.125	0.1428	0.1666	0.2000
Difference of Digree	3	2	1	0
Coupling strength	0.2500	0.3333	0.500	1.000

The normalized circuit equation of these network models is given by the following equations.

$$\begin{cases} \frac{dx}{d\tau} = z_n, \\ \frac{dy}{d\tau} = \alpha\gamma y_n - \alpha f(y_n - z_n) - \alpha\delta \sum_{k \in S_n} (y_n - y_k), \\ \frac{dz}{d\tau} = f(y_n - z_n) - x_n. \end{cases} \quad (7)$$

The parameter δ corresponds the coupling strength between the circuits. We set the parameter as α = 0.5, β = 20.0 and γ = 0.5.

IV. Simulation results

We define synchronization as the following Eq. (8). Synchronization rate is ratio of number of synchronization to all calculate point.

$$|y_j - y_i| < 0.03 \qquad (i, j = 1, 2, \cdots, 20) \quad (8)$$

In Fig. 3, we sets of only coupling strength with degree difference of 5. Simillaly, in Fig. 4, we sets of only coupling strength with degree different of 6.

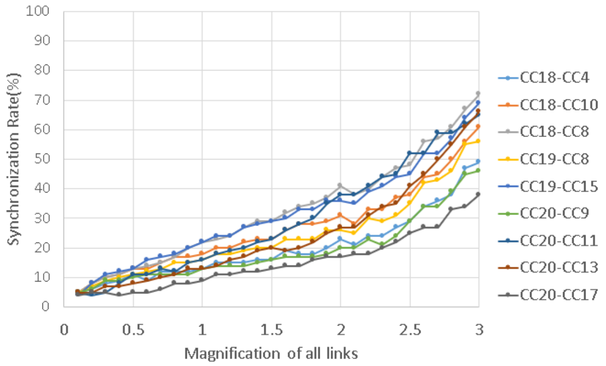

Fig. 3: Synchronization rate of 5 degrees coupling.

In Fig. 3, lowest synchronization rate is 45% and highest synchronization rate is 91%.

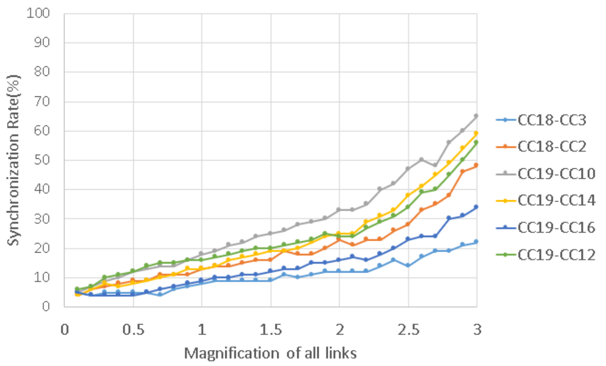

Fig. 4: Synchronization rate of 6 degrees coupling.

In Fig. 4, lowest synchronization rate is 27% and highest synchronization rate is 72%. In this result, even if several couplings have same coupling strength, synchronization is different.

We estimate this reason is indirectly connection. Next, we focus on CC18-CC19. Some couplings were multiplied magnification in network.

Figure 5 shows synchronization of CC18-CC19 when we change the coupling strength to CC1-CC6, CC6-CC12 , CC5-CC9 , CC11-CC20, CC10-CC19 and CC4-CC5.

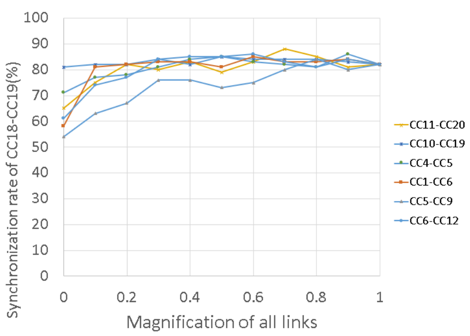

Fig. 5: Synchronization when multiplied some magnification to coupling.

In Fig. 5, some synchronization was greatly changed when we multiplied low magnification to coupling strength. These couplings are indirectry couplings for CC18-CC19, and strongly couplings. Therefore, we estimate that synchronization are involved in the synchronization of CC18-CC19.

When some couplings affected to CC18-CC19, coupling strength is 0 to 0.1. Next, we investigate synchronization of CC18-CC19 when each coupling strength is changed. We confirm coupling which changed CC18-CC19. When synchronization rate reaches 50% to 70%, we judged that synchronization has changed.

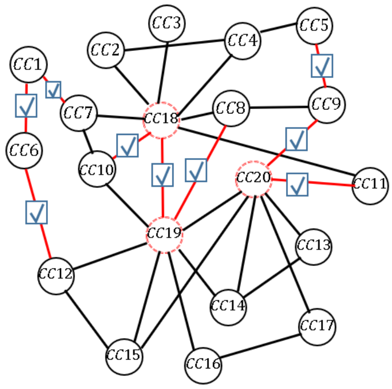

Fig. 6: Changed coupling.

In Fig 6, synchronization becomes constant on average when indirectly couplings have a certain level of the coupling strength. When some couplings open or extremely weak coupling strength, synchronization is associated with some connect will be weak. At the same time, synchronization was not changed when we change coupling strength in other indirectly coupling.

Next, we investigate this reason. When coupling strength was changed, especially we open coupling, this phenomena was caused. Therefore, link was reconnected which effect too many coupling at other nodes. First, we choice too effective links from all links. We choose three links which changes too many nodes and number of links is not same. Then, link are reconnected on one side. In this study, CC9-CC20, CC7-CC18 and CC6-CC12 are chosen. These links are reconnected to CC5-CC20, CC7-CC2 and CC1-CC12. Each results is shown at Fig. 7 to Fig. 9. At this time, a number of links are changed. However, coupling strength was not changed. We compare this result and original network result.

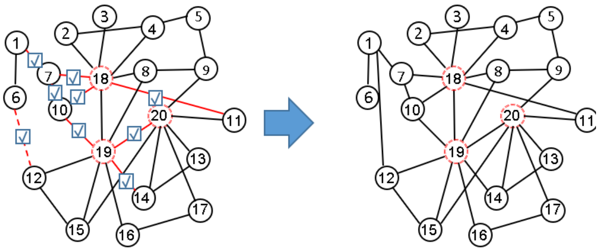

Fig. 7: Reconnect CC6-CC12 to CC1-CC12.

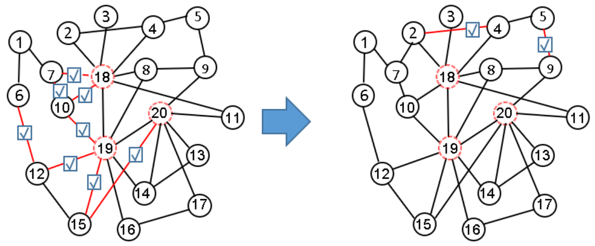

Fig. 8: Reconnect CC7-CC18 to CC7-CC2.

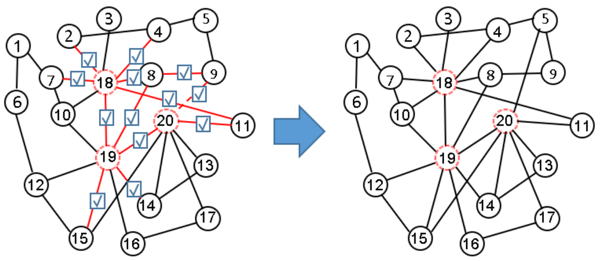

Fig. 9: Reconnect CC9-CC20 to CC5-CC20.

In this result, change is return ed. We understand that network form effect to all synchronization. However, the phenomenon does not change when the network become nearby form. Next, we investigate the effect of the coupling strength. In this study, the coupling strength was changed. the coupling strength was equalized and compared to original network result. All coupling strength average value is 0.3953. When we use this value and multiplied value 3.0, all couplings synchronization rate become 99%. Therefore this value was decreased to 1.0. We set rank of synchronization rate and compare from original network result. In this table, "sync" mean synchronization rate.

TABLE II: Compare equalized and original network.

All 0.3958			Original Network				All 0.3958			Original Network		
Rank	Link	Sync	Rank	Link	Sync		Rank	Link	Sync	Rank	Link	Sync
5	1-6	53	2	1-6	99	Average differece of Rank	17	10-19	31	20	10-19	65
29	1-7	22	14	1-7	83		18	11-18	31	17	11-18	74
2	2-4	81	3	2-4	99	4.875	13	11-20	38	9	11-20	93
26	2-18	24	28	2-18	48		15	12-15	35	8	12-15	98
30	3-18	22	32	3-18	22		22	12-19	27	24	12-19	57
3	4-5	58	4	4-5	99	Median difference of Rank	4	13-14	56	7	13-14	99
28	4-18	23	29	4-18	48		19	13-20	30	16	13-20	81
21	5-9	27	15	5-9	82	7.5	20	14-19	30	18	14-19	70
31	6-12	22	13	6-12	84		8	14-20	51	11	14-20	89
12	7-10	38	5	7-10	99		9	15-19	50	21	15-19	63
24	7-18	25	27	7-18	52	Total difference of Rank	14	15-20	37	22	15-20	62
6	8-9	53	6	8-9	99		1	16-17	82	1	16-17	100
10	8-18	44	19	8-18	65	156	32	16-19	20	31	16-19	39
16	8-19	33	23	8-19	60		27	17-20	24	30	17-20	43
25	9-20	25	26	9-20	55		23	18-19	27	12	18-19	86
11	10-18	42	25	10-18	56		7	19-20	52	10	19-20	92

In this result, the average difference of rank is 4.875. And some coupling have too many differences. We understand that effect of the coupling strength is important too.

V. Conclusions

In this study, we investigated the influence of path by using a scale-free coupled chaotic network by changing the couplings strength. First, we compare synchronization on same coupling strength in the network. Next, we compare some synchronization when coupling strength was changed some values.

In this result, we found that the strength of synchronization is different even with the same coupling strength. We tried investigated reasons of different synchronization. We investigate one synchronization when some coupling strength was changed some values. At this time, synchronization was weakened when some coupling strength was weakened. We consider some reason. However, because several situations contradicted each other, we could not clarify the reason. In addition, we find internet form is important.

In future work, we need to clarify the phenomenon of this time. Furthermore, we would like to verify only circuit simulation. If we simulate society or nature network, we need to investigate characteristic of the network. We need to consider deciding corresponding parameter, defining criteria, configuring network.

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