Synchronization Phenomena in Coupled Nonlinear Oscillators Chains with Hourglass Structure

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***Abstract*—In this study, we investigate synchronization phenomena from circuit network, which is composed of several numbers of van der Pol oscillator chains. We use van der Pol oscillators which are coupled by resistors. We investigate how synchronization phenomena changes by changing the number of van der Pol oscillators. From the computer simulations, we observe relatively interesting synchronization phenomena.**

I. Introduction

Synchronization phenomena are the most familiar phenomena that exist in nature and it has been studied in various fields, such as in electrical, in mechanics, in biological systems, basically everywhere. Synchronization phenomena are observed everywhere in our life. For example, we can confirm flashing firefly lights, metronome, periodic swinging of candle flames, gate patterns of four-leg animals, beating rhythm of the heart and so on. Among them, van der Pol oscillators can observe phenomena similar to natural phenomena. Therefore we have been interested in coupled oscillators which synchronization phenomena from circuit network.

In this study, we investigate synchronization phenomena from a circuit network, which is composed of several numbers of van der Pol oscillator chains. The circuit model of van der Pol oscillator is shown in Fig. 1. This circuit consists of an inductor, a capacitor and a nonlinear resistance. From the computer simulations, we observe relatively interesting synchronization phenomena. We focus on synchronization phenomena in the coupled nonlinear circuits.

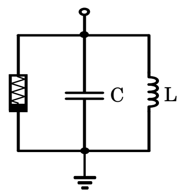

Fig. 1. Circuit of van der Pol.

II. Previous study

Figure 2 shows the previous study circuit model. Bottom, middle and top are coupled with oscillators by resistors r. Bottom several numbers of oscillator are coupled by resistors R_i. Further, top several numbers of oscillator are coupled to ground via resistors R_a through inductors. Middle oscillators are not coupled with oscillators located in the horizontal direction but coupled vertically. In this network, we coupled the oscillators of bottom several numbers of oscillator in the chains to tend to produce in-phase synchronizations. Further, we coupled the oscillators of the top several numbers of oscillator in the chains to tend to produce anti-phase synchronizations. Middle oscillators are coupled with bottom and top several numbers of oscillator. However, they are not coupled with the other chains.[1]

We define the bottom several numbers of oscillator as Osc_{11}, Osc_{12}, and Osc_{1m} from the left. Further the middle several numbers of oscillator as Osc_{21}, Osc_{22}, and Osc_{2m} from the left and the top several numbers of oscillator as Osc_{31}, Osc_{32}, and Osc_{3m} from the left.

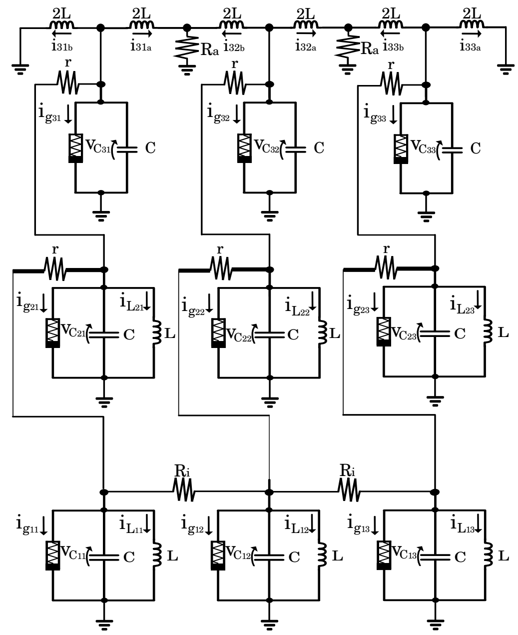

Fig. 2. Previous study system model.

First, we assume that the $v-i$ characteristics of the nonlinear resistor in each oscillator are given by the follows:

$$i_{gn} = -g_1 v_n + g_3 v_n^3 \tag{1}$$

where $g_1, g_3 > 0$.

III. System model

Next in this study, we investigate to change the number of middle oscillators. Figure 3 shows the circuit model. This model is named “Hourglass Structure”. The difference from the previous study is that it has one middle oscillator.

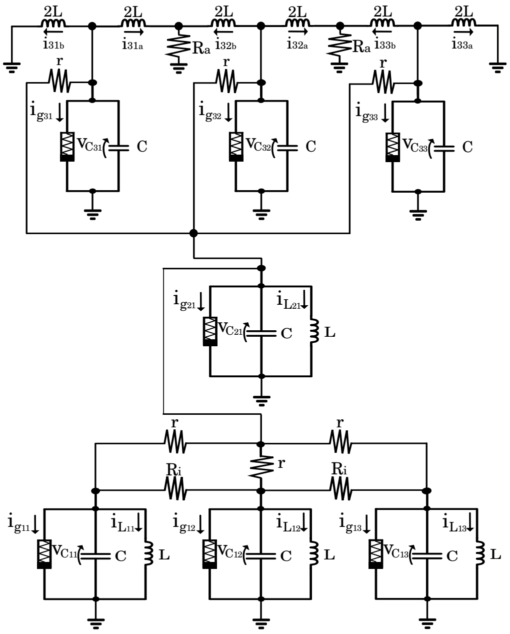

Fig. 3. Hourglass Structure.

The circuit equations are given as follows:

(1) Bottom oscillators :

$$\begin{cases} C\dfrac{dv_{c1j}}{dt} + i_{L1j} + i_{g1j} + i_{Ri} + i_r = 0, \\ C\dfrac{dv_{c1l}}{dt} + i_{L1l} + i_{g1j} + i_{Ri} + i_{Ri} + i_r = 0, \\ L\dfrac{di_{L1k}}{dt} = v_{c1k}. \end{cases} \tag{2}$$

(2) Middle oscillators :

$$\begin{cases} C\dfrac{dv_{c21}}{dt} + i_{L21} + i_{g21} + 2mi_r = 0, \\ L\dfrac{di_{L21}}{dt} = v_{c21}. \end{cases} \tag{3}$$

(3) Top oscillators :

$$\begin{cases} C\dfrac{dv_{c3k}}{dt} + i_{g3k} + i_r + i_{3ka} + i_{3kb} = 0, \\ 2L\dfrac{di_{L31a}}{dt} + R_a(i_{31a} + i_{32b}) = v_{c31}, \\ 2L\dfrac{di_{L31b}}{dt} = v_{c31}, \\ 2L\dfrac{di_{L3la}}{dt} + R_a(i_{3la} + i_{3(l+1)b}) = v_{c3l}, \\ 2L\dfrac{di_{L3lb}}{dt} + R_a(i_{3(l-1)a} + i_{3lb}) = v_{c3l}, \\ 2L\dfrac{di_{L3ma}}{dt} = v_{c3m}, \\ 2L\dfrac{di_{L3mb}}{dt} + R_a(i_{3(m-1)a} + i_{3mb}) = v_{c3m}. \end{cases} \tag{4}$$

By using the following variables and parameters:

$$\begin{cases} t = \sqrt{LC}\tau, v_{kj} = \sqrt{\dfrac{g_1}{g_3}}x_{kj} \\ i_{kj} = \sqrt{\dfrac{g_1 C}{g_3 L}}y_{kj}, \varepsilon = g_1\sqrt{\dfrac{C}{L}} \\ \alpha_a = R_a\sqrt{\dfrac{C}{L}}, \alpha_i = \dfrac{1}{R_i}\sqrt{\dfrac{L}{C}}, \beta = \dfrac{1}{r}\sqrt{\dfrac{L}{C}}. \end{cases} \tag{5}$$

The normalized equation of this circuit is given as follows:

(1) Bottom oscillators :

$$\begin{cases} \dot{x_{1j}} = \varepsilon x_{1j}(1 - {x_{1j}}^2) - y_{1j} \\ \qquad -\alpha_i(x_{1j} - x_{1l}) - \beta(x_{1j} - x_{21}), \\ \dot{x_{1l}} = \varepsilon x_{1l}(1 - {x_{1l}}^2) - y_{1l} \\ \qquad +\alpha_i(x_{1(l-1)} - 2x_{1l} + x_{1(l+1)}) - \beta(x_{1l} - x_{21}), \\ \dot{y_{1k}} = x_{1k}. \end{cases} \tag{6}$$

(2) Middle oscillators :

$$\begin{cases} \dot{x_{21}} = \varepsilon x_{21}(1 - {x_{21}}^2) - y_{21} \\ \qquad +\beta(x_{11} + ... + x_{1m} - 2mx_{21} + x_{31} + ... + x_{3m}), \\ \dot{y_{21}} = x_{21}. \end{cases} \tag{7}$$

(3) Top oscillators :

$$\left\{\begin{aligned}
\dot{x_{3k}} &= \varepsilon x_{3k}(1 - {x_{3k}}^2) \\
&\quad -(y_{3ka} + y_{3kb}) - \beta(x_{3k} - x_{21}), \\
\dot{y_{31a}} &= \frac{1}{2}\{x_{31} - \alpha_a(y_{31a} + y_{32b})\}, \\
\dot{y_{31b}} &= \frac{1}{2}x_{31}, \\
\dot{y_{3la}} &= \frac{1}{2}\{x_{3l} - \alpha_a(y_{3la} + y_{3(l+1)b})\}, \\
\dot{y_{3lb}} &= \frac{1}{2}\{x_{3l} - \alpha_a(y_{3(l-1)a} + y_{3lb})\}, \\
\dot{y_{3ma}} &= \frac{1}{2}x_{3m}, \\
\dot{y_{3mb}} &= \frac{1}{2}\{x_{3m} - \alpha_a(y_{3(m-1)a} + y_{3mb})\}.
\end{aligned}\right. \tag{8}$$

$(j = 1, m \ \cdot \ k = 1, 2, ..., m \ \cdot \ l = 2, 3, ..., m - 1)$.

IV. Simulation results

(A) Case of previous study

Figures 4 and 5 show the computer simulation results for the case of the previous study model. The circuit parameters are chosen as ε = 0.10, α_a = α_i = 0.50 and β = 0.02.

The bottom three oscillators are become in-phase synchronization and the top three oscillators are become anti-phase synchronization.

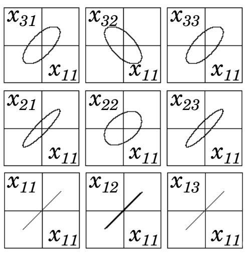

Fig. 4. Computer simulation results (phase shift) for previous study.

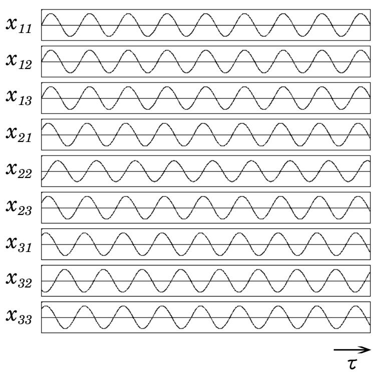

Fig. 5. Computer simulation results (time waveforms) for previous study.

(B) Case of hourglass structure (m = 3)

Figures 6 and 7 show the computer simulation results for the case of hourglass structure. The circuit parameters are chosen at the same.

In comparison with (A), the phase shift with Osc_{11} is approaching in-phase synchronization or anti-phase synchronization.

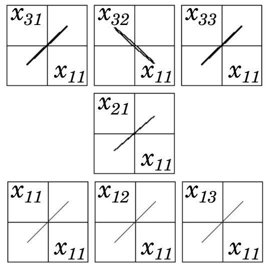

Fig. 6. Computer simulation results (phase shift) for hourglass structure (m = 3).

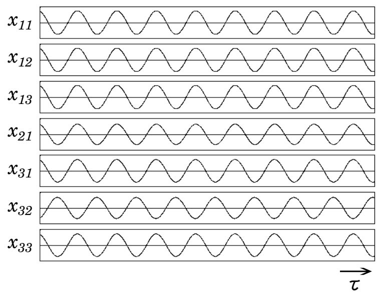

Fig. 7. Computer simulation results (time waveforms) for hourglass structure (m= 3).

(C) Case of hourglass structure (m = 4)

Figures 8 and 9 show the computer simulation results for the case of hourglass structure. The circuit parameters are chosen at the same.

In comparison with (B), we increase one oscillator to the top and bottom. As (A) and (B), the bottom oscillators are in-phase synchronization and the top oscillators are anti-phase synchronization. However, in comparison with (B), the top oscillators are not in-phase synchronization and anti-phase synchronization with Osc_{11}.

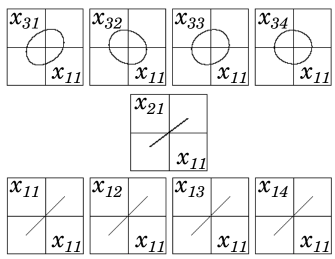

Fig. 8. Computer simulation results (phase shift) for hourglass structure (m = 4).

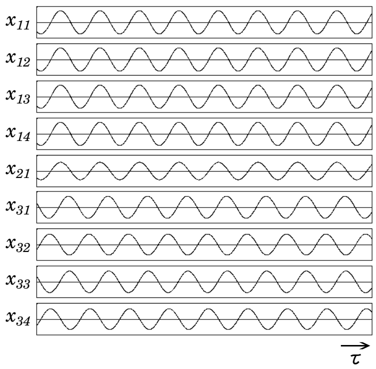

Fig. 9. Computer simulation results (time waveforms) for hourglass structure (m= 4).

From Figs. 6 and 8 there is the difference in the synchronization phenomena of the top oscillators. The top oscillators of all results are anti-phase synchronization. However, looking at the result of phase shift with Osc_{11}, when the value of m is an odd number, the phase shift with Osc_{11} is approaching the in-phase synchronization or anti-phase synchronization, and when the value of m is an even number, the different synchronization was observed in Osc_{11} and top oscillators.

V. Conclusions

In this study, we have investigated synchronization phenomena observed by circuit networks, which are composed of several numbers of van der Pol oscillator chains. By the computer simulations, we could obtain of results that the bottom oscillators observe in-phase synchronization and the top oscillators observe anti-phase synchronization. Further, we could observe synchronization phenomena by changing with the value of m.

In the future works, we will investigate synchronization phenomena using other parameters. Further, when the value of m is an odd number or an even number, the synchronization phenomena occur in the Osc_{21} of the middle oscillator and guess some cause. Therefore, we investigate this case in detail. Further, in this study, we have investigated oscillators connected symmetrically to left and right, but we would like to investigate circuit models that are connected asymmetrically to left and right by increasing the number of oscillators in the middle.

References

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[4] T. Matsunashi, D. Nariai, Y. Uwate and Y. Nishio, "Synchronization Phenomena in a Ring of van der Pol Oscillators Coupled by Time-Varying Resistor," Proceedings of RISP International Workshop on Nonlinear Circuits, Communications and Signal Processing (NCSP'18), pp. 168-171, Mar. 2018.