1-15

Synchronization Phenomena in Coupled Nonlinear Oscillators with Hourglass Structure

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1. Introduction

In this study, we investigate synchronization phenomena of van der Pol oscillators of hourglass structure by changing the coupling strengths. We use van der Pol oscillators which are coupled by resistors.

2. System Model

The circuit model of the hourglass structure using van der Pol oscillators is shown in Fig. 1.

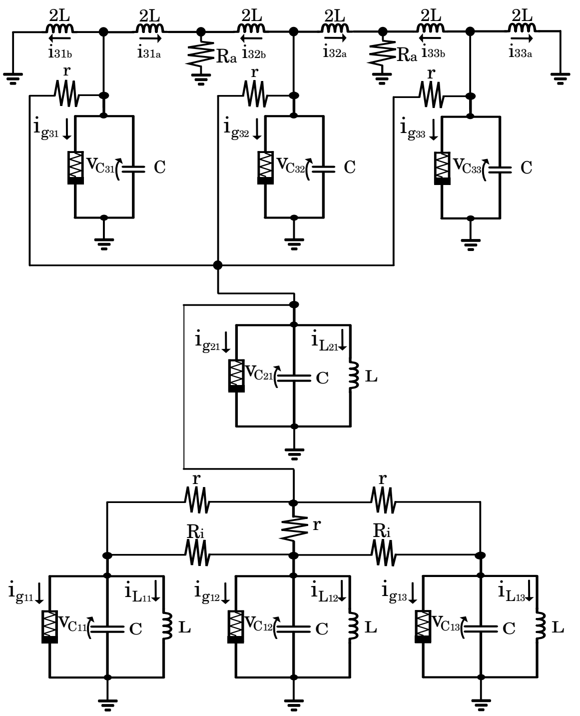

Figure 1: Circuit model of Hourglass Structure.

The normalized circuit equations of this circuit equations are given by the following equations.

(1) Bottom oscillators:

$$\begin{cases} \dot{x_{11}} = \varepsilon x_{11}(1 - {x_{11}}^2) - y_{11} \\ \qquad -\alpha_i(x_{11} - x_{12}) - \beta(x_{11} - x_{21}) \\ \dot{y_{11}} = x_{11} \\ \dot{x_{12}} = \varepsilon x_{12}(1 - {x_{12}}^2) - y_{12} \\ \qquad +\alpha_i(x_{11} - 2x_{12} + x_{13}) - \beta(x_{12} - x_{21}) \\ \dot{y_{12}} = x_{12} \\ \dot{x_{13}} = \varepsilon x_{13}(1 - {x_{13}}^2) - y_{13} \\ \qquad -\alpha_i(x_{13} - x_{12}) - \beta(x_{13} - x_{21}) \\ \dot{y_{13}} = x_{13} \end{cases} \tag{1}$$

(2) Middle oscillators:

$$\begin{cases} \dot{x_{21}} = \varepsilon x_{21}(1 - {x_{21}}^2) - y_{21} \\ \qquad +\beta(x_{11} + x_{12} + x_{13} - 6x_{21} + x_{31} + x_{32} + x_{33}) \\ \dot{y_{21}} = x_{21} \end{cases} \tag{2}$$

(3) Top oscillators:

$$\begin{cases} \dot{x_{31}} = \varepsilon x_{31}(1 - {x_{31}}^2) - (y_{31a} + y_{31b}) - \beta(x_{31} - x_{21}) \\ \dot{y_{31a}} = 0.5x_{31} - \alpha_a(y_{31a} + y_{32b}) \\ \dot{y_{31b}} = 0.5x_{31} \\ \dot{x_{32}} = \varepsilon x_{32}(1 - {x_{32}}^2) - (y_{32a} + y_{32b}) - \beta(x_{32} - x_{21}) \\ \dot{y_{32a}} = 0.5x_{32} - \alpha_a(y_{32a} + y_{33b}) \\ \dot{y_{32b}} = 0.5x_{32} - \alpha_a(y_{31a} + y_{32b}) \\ \dot{x_{33}} = \varepsilon x_{33}(1 - {x_{33}}^2) - (y_{33a} + y_{33b}) - \beta(x_{33} - x_{21}) \\ \dot{y_{33a}} = 0.5x_{33} \\ \dot{y_{33b}} = 0.5x_{33} - \alpha_a(y_{32a} + y_{33b}) \end{cases} \tag{3}$$

The parameter β corresponds the coupling strength between the circuits.

3. Simulation results

In this study, we change the coupling strengths of the resistors connected to the middle van der Pol oscillator.

Figure 2 shows the computer simulation results. The circuit parameters are chosen as $\varepsilon = 0.10$, $\alpha_i = \alpha_a = 0.5$, and $\beta = 0.02$.

In this result, we have confirmed that all circuits are synchronized when the coupling strengths are $\beta = 0.02$. On the other hand, we have confirmed anti-phase synchronization only at x_{32} while nearly all circuits are synchronized with in-phase.

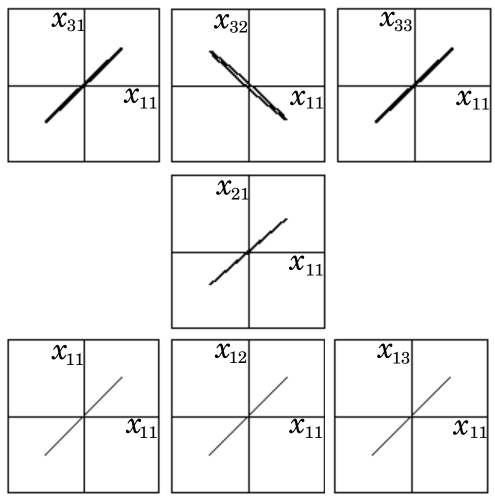

Figure 2: Computer simulation results (phase shift)

4. Conclusion

In this study, we investigated synchronization phenomena of the hourglass structure using van der Pol oscillators.

As our future works, we will confirm the synchronization phenomena by changing the coupling strengths of the resistors connecting the top and bottom van der Pol oscillators and increasing the number of van der Pol oscillators in the middle.