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Synchronization in Coupled van der Pol Oscillators Containing Three Oscillators with Star Structure Connected to Another Oscillator

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1. Introduction

In this study, we propose a new type of coupled van der Pol oscillators: three oscillators with star structure connected to another oscillator. By carring out computer simulations, the relationship of the model between synchronization phenomena and coupling strengths is investigated.

2. System Model

Figure 1 shows a system model consisting of van der Pol oscillators. Three van der Pol oscillators are coupled as star struture that connected to another oscillator via resistor r. We investigate synchronization phenomena by changing the coupling strength of the resistors r_2, r_3, r_4.

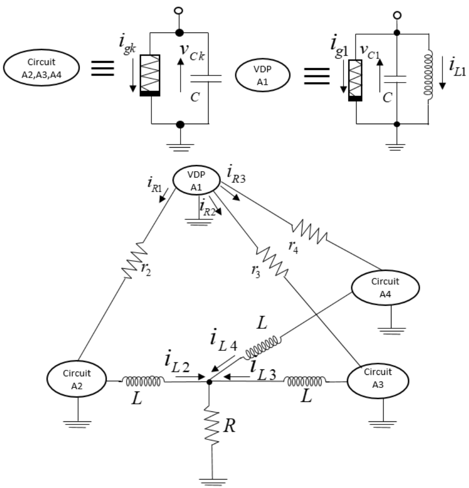

Figure 1: Circuit model.

The characteristics of the nonlinear resistors are defined as follows:

$$i_{gk} = -g_1 v_k + g_3 v_k^3. \tag{1}$$

the normalized circuit equations of VDP-A1 are given as follows:

$$\begin{cases} \dfrac{dx_1}{d\tau} = \alpha\left(x_1 - \dfrac{1}{3}x_1^3\right) - y_1 - \beta_n\left(3x_1 - \displaystyle\sum_{m=2}^{4} x_m\right) \\ \dfrac{dy_1}{d\tau} = x_1. \end{cases} \tag{2}$$

the normalized circuit equations of Circuit-A2, Circuit-A3, Circuit-A4 are given as follows:

$$\begin{cases} \dfrac{dx_k}{d\tau} = \alpha\left(x_k - \dfrac{1}{3}x_k^3\right) - y_k + \beta_n(x_1 - x_k) \\ \dfrac{dy_k}{d\tau} = x_k - \gamma \displaystyle\sum_{m=2}^{4} y_m, \end{cases} \tag{3}$$

$$(n, k = 2, 3, 4).$$

where parameters α, β, and the γ denote nonlinearity, the resistors r, and the resistor R, respectively.

3. Simulation results

For the computer simulations, we calculates Eqs. (2)-(3) by using Runge-Kutta method with the step size $h = 0.05$. When the parameters are fixed as $\alpha = 0.1, \gamma = 0.006$, we control synchronization phenomena of this circuit model by changing the coupling strengths β_2, β_3, β_4.

First, in the case of parameters β_2, β_3, β_4 are set to 0.015, Fig. 2 shows the attractor of each oscillator. Next, we sightly increase the parameters β_2, β_3, β_4 to the same value as 0.017, all fours oscillators become in-phase as Fig. 3. When we only change β_2 to 0.005, the oscillator of Circuit A2 becomes anti-phase with other oscillators, when we change β_2, β_3 to 0.0005, oscillator of VDP-A1 becomes in-phase with oscillator of Circuit A2 and the oscillators of Circuit A2, A3, A4 become 3-phase synchronization. These results are shown in Figs. 4-5.

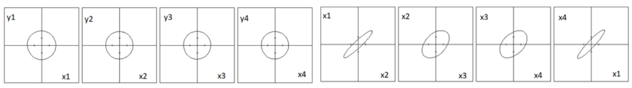

Figure 2: Phase differences ($\beta_2 = \beta_3 = \beta_4 = 0.015$).

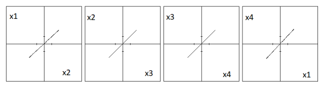

Figure 3: Phase differences ($\beta_2 = \beta_3 = \beta_4 = 0.017$).

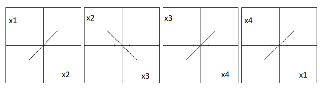

Figure 4: Phase differences ($\beta_2 = 0.005$, $\beta_3 = \beta_4 = 0.015$).

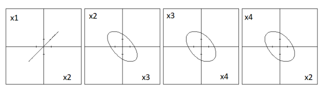

Figure 5: Phase differences ($\beta_2 = \beta_3 = 0.0005$, $\beta_4 = 0.015$).

4. Conclusion

This study mainly concerned about synchronization phenomena, especially about phase difference between the oscillators when the coupling strengths are changed. In the next step, it is necessary to increase the number of oscillators and complete the circuit experimental, and it is expected to bring us more interesting phenomena and we can find a good solutions for larger coupled systems.