Synchronization Phenomena in Coupled Oscillators Containing Star Structure Connected to Another Oscillator with Different Coupling Strength

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***Abstract*—In this study, synchronization phenomena in coupled oscillators containing star structure connected to another oscillator is investigated. We focus on the phase different between two oscillators when coupling strength is changed. Furthermore, we also investigate the effect of the initial condition. By using computer simulations, we observe synchronization phenomena of the system.**

I. Introduction

Synchronization phenomena are observed everywhere in our world. For example, firefly luminescence, birds and frogs crying, human aplause and so on. It has a long history of research and its applications can be widely found in many fields of science, such as chemical, physical, biological, and also social systems. Therefore, investigation of synchronization phenomena has become an important issue.

The van der Pol oscillator is the symple circuit. It consists of resistor, inductor, capacitor and nonlinear resistor. Synchronization phenomena of van der Pol oscillator is the one of nolinear phenomena that can be observed similar natural phenomena by controling frequencies.

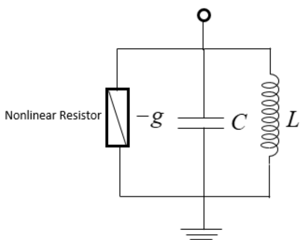

Fig. 1. van der Pol oscillator.

In this study, we proposed a new type of coupled van der Pol oscillators system: Star structure connected to another oscillator. By carring out computer simulations, relationship of the model between synchronization phenomena and coupling strength is investigated.

II. Circuit model

The proposed circuit is shown in Fig. 2. We used three van der Pol oscillator coupled as star structure that connected another oscillator via resistor r. We investigate synchronization phenomena by changing coupling strength of resistor R.

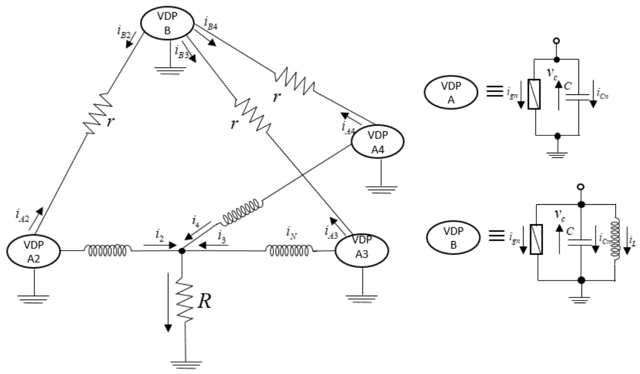

Fig. 2. Circuit model.

The circuit equations of VDP-B are given as folows:

$$\begin{aligned}
&-i_{g1} - i_{c1} - i_{L1} = i_{B2} + i_{B3} + i_{B4} \\
&L\frac{di_{L1}}{dt} = v_1 \\
&v_1 - v_2 = r(i_{B2} - i_{A2}) \\
&v_1 - v_3 = r(i_{B3} - i_{A3}) \\
&v_1 - v_4 = r(i_{B4} - i_{A4}),
\end{aligned} \tag{1}$$

where v_1 denote the voltage of oscillator VDP-B.
The circuit equations of VDP-A are given as follows:

$$\begin{aligned}
&-i_{gk} - i_{ck} = i_{Ak} + i_k \\
&v_k - v_{Lk} = R\sum_{n=2}^{4} i_{Ln}. \\
&(k = 2, 3, 4)
\end{aligned} \tag{2}$$

Nonlinear resistor defined as follows:

$$i_{gn} = -g_1 v_n + g_3 v_n^3. \tag{3}$$

By changing the variables and parameter:

$$
\begin{aligned}
&t=\sqrt{LC}\tau, v_n=\sqrt{\frac{g_1}{3g_3}}x_n,\\
&i_n=\sqrt{\frac{g_1C}{3g_3L}}y_n, \alpha=g_1\sqrt{\frac{L}{C}},\\
&\beta=\frac{1}{r}\sqrt{\frac{L}{C}}, \gamma=R\sqrt{\frac{C}{L}}.
\end{aligned}
\tag{4}
$$

The normalized circuit equation of VDP-B are given as follows:

$$
\begin{cases}
\dfrac{dx_1}{d\tau}=\alpha(x_1-\dfrac{1}{3}x_1^3)-y_1+\beta(3x_1-\displaystyle\sum_{n=2}^{4}x_n)\\
\dfrac{dy_1}{d\tau}=x_1.
\end{cases}
\tag{5}
$$

The normalizaed circuit equation of VDP-A are given as follows:

$$
\begin{cases}
\dfrac{dx_n}{d\tau}=\alpha(x_n-\dfrac{1}{3}x_n^3)-y_n-\beta(x_1-x_n)\\
\dfrac{dy_n}{d\tau}=x_n-\gamma\displaystyle\sum_{n=2}^{4}y_n,
\end{cases}
\tag{6}
$$

where parameters α, β, γ denote the coupling strengh of the inductor, resistors r and resistor R.

III. Synchronization Phenomena

For the computer simulations, we calculates Eqs.(2)-(5) by using the Runge-Kutta method with the step size $h = 0.05$. We show the simulation results when the parameters of the circuit model are fixed as $\alpha = 0.01$, $\beta = 0.01$ and $\gamma = 0.007$.

Figure 3 shows the attractor of each oscillator with the horizontal axis is the voltage and the vertical axis is the electric current of each oscillator.

First, we investigate the effect of different the initial condition. As simulation results are shown in Fig. 4, we obtain that, by changing initial values, 2 oscillators of VDP-A can become in-phase.

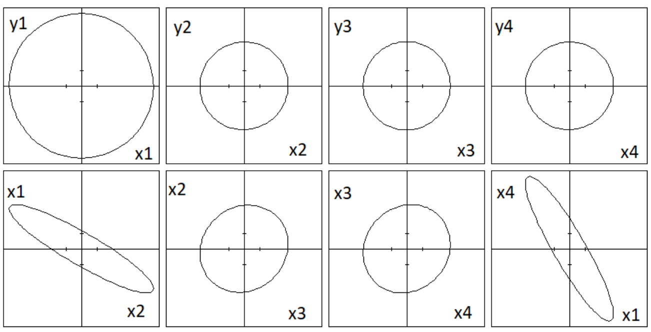

Fig. 3. Attractor between adjacent oscillators ($\alpha = 0.01$, $\beta = 0.01$, $\gamma = 0.007$).

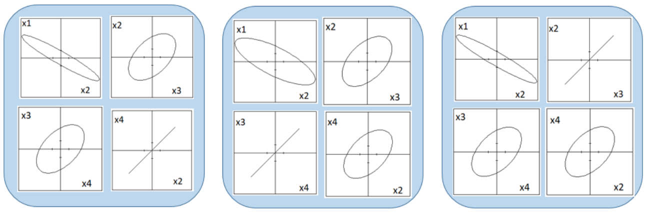

Fig. 4. Phase difference ($\alpha = 0.01$, $\beta = 0.01$, $\gamma = 0.007$).

Next, we decrease the values of parameter γ from 0.007 to 0.005 and continue conducting computer simulation. In this case, all four oscillatos become synchronization with second, third and fourth oscillator of VDP-A are in-phase and only first oscillator of VDP-B is anti-phase. The result of simulation is shown in Fig. 5.

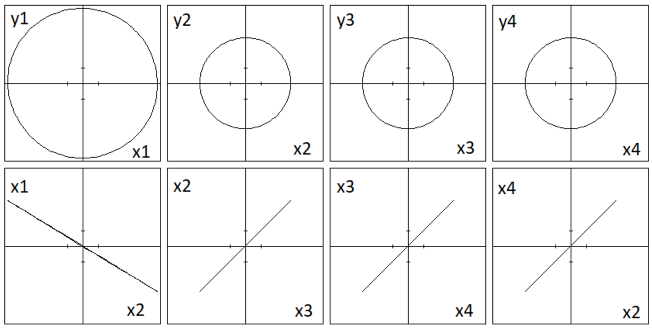

Fig. 5. Phase difference ($\alpha = 0.01$, $\beta = 0.01$, $\gamma = 0.005$).

Figure 6 shows the computer simulation results of the phase difference when parameter γ is changed from 0.001 to 0.009 with step size 0.001. In this case, we can obtain that, when value of parameter γ is smaller than 0.005, second, third, fourth oscillator of VDP-A are in-phase and only first oscillator of VDP-B is anti-phase.

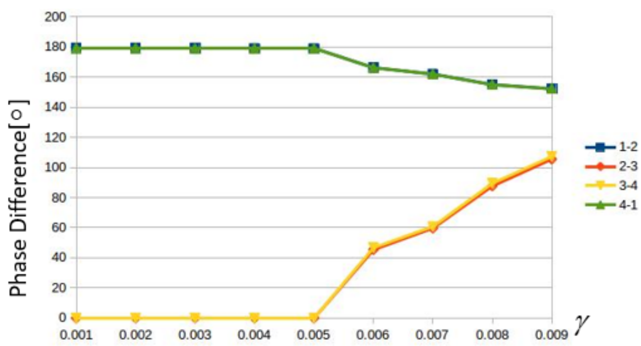

Fig. 6. Phase difference ($\alpha = 0.01$, $\beta = 0.01$).

Therefore, we can control synchronization phenomena by changing initial values or the coupling strengths.

IV. CONCLUSIONS

In this study, we have investigated the synchronization phenomena in coupled oscillators containing star structure connected to another oscillator. By computer simulation, we have observed the synchronization phenomena by changing initial values and coupling strength. In the case of chaging initial values, in three oscillators of VDP-A, two oscillators can become in-phase. By decreasing coupling strength, all four oscillators can become synchronization with oscillators of VDP-A are in-phase and only oscillator of VDP-B is anti-phase.

In the future, we experiment the proposed circuit model.

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