

Synchronization Phenomena of Chaotic Circuit Networks with Distributed Hub

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Abstract—In this study, we focus on network structure with a hub which is consisted of Scale-free network. We consider the synchronization of coupled chaotic circuit network whose hub nodes are split two nodes and we investigate two network models. In one of them, we replace a hub node with two distributed nodes connected by a positive resistor. In another, we replace a hub node with two distributed nodes connected by a negative resistor. From the simulation results, we can confirm that connection between hubs plays an important role in network structure.

I. INTRODUCTION

Synchronization phenomena can be observed everywhere in our life. For example, we can confirm metronome, flashing firefly lights, beating rhythm of the heart and so on. Especially, synchronization phenomena of oscillatory network are interesting. In addition, complex networks attract attention from various fields. The feature of networks is characterized by the degree distribution, the path length and the clustering coefficient.

Numerous researches on complex networks have been made so far [1]-[7]. Especially, there are many studies related with hub nodes and these are very interesting. Because, in the real world, large-scale complex networks often include huge nodes with a large number of edges, and such “hub” nodes play key roles in the networks.

In this study, we pay our attention on the behavior of the networks whose hub nodes are split into several nodes. Namely, when such a hub node is split into several nodes, how other nodes connected to the hub node change their behavior? Furthermore, when such split hub nodes agree with or oppose to each other, how the whole network behaves to cooperate or to compete? We construct the complex networks by using coupled chaotic circuits and evaluate the behavior by using synchronization rate between a pair of the chaotic circuits. In the first model, we replace a hub node with two distributed hub nodes connected by a positive resistor. In the second model, we replace a hub node with two distributed hub nodes connected by a negative resistor. We evaluate these models by synchronization rate and compare their behaviors. In distributed hubs connected by a positive resistor, we compare the synchronization phenomena of the network whose hub node

replaced with two distributed nodes and the network before splitting a hub. Further, we investigate the effect of the network structure around the hub node on the behavior of the whole network.¹

II. CIRCUIT MODEL

The chaotic circuit model which proposed by Shinriki *et al.* [9][10] is shown in Fig. 1. This circuit consists of a negative resistor, an inductor, two capacitors and dual-directional diodes. In this study, we use a network in which each node is replaced by a chaotic circuit. The circuit equation is shown in Eq. (1).

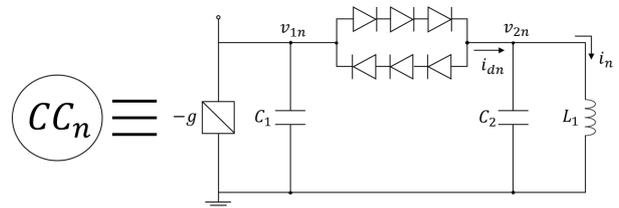


Fig. 1. Circuit model.

$$\begin{cases} L \frac{di_n}{dt} = v_{2n} \\ C_1 \frac{dv_{1n}}{dt} = gv_{1n} - i_{dn} - \frac{1}{R} \sum_{k \in S_n} (v_{1n} - v_{1k}) \\ C_2 \frac{dv_{2n}}{dt} = -i_n - i_{dn}, \end{cases} \quad (1)$$

where $n = 1, 2, 3, \dots, 10$ and S_n is the set of nodes which are directly connected to the node n . The characteristic of nonlinear resistor which consists of dual diodes is following Eq. (2).

¹The extended version of this study is being reviewed for ISCAS'18 [8].

$$i_{dn} = \begin{cases} G_d(n_{1n} - v_{2n} - V) & (v_{1n} - v_{2n} > V) \\ 0 & (|n_{1n} - v_{2n}| \leq V) \\ G_d(n_{1n} - v_{2n} + V) & (v_{1n} - v_{2n} < -V). \end{cases} \quad (2)$$

By changing the variables and parameters,

$$\begin{cases} i_n = \sqrt{\frac{C_2}{L}} V x_n, & v_{1n} = V y_n, & v_{2n} = V z_n \\ \alpha = \frac{C_2}{C_1}, & \beta = \sqrt{\frac{L}{C_2}} G_d, & \gamma = \sqrt{\frac{L}{C_2}} g, \\ \delta = \frac{1}{R} \sqrt{\frac{L}{C_2}}, & t = \sqrt{L C_2} \tau, & \text{"."} = \frac{d}{dt}, \end{cases} \quad (3)$$

the normalized equations of chaos circuits are given as follows:

$$\begin{cases} \dot{x} = z_n \\ \dot{y} = \alpha \gamma y_n - \alpha \beta f(y_n - z_n) - \alpha \delta \sum_{k \in S_n} (y_n - y_k) \\ \dot{z} = \beta f(y_n - z_n) - x_n. \end{cases} \quad (4)$$

The parameter δ corresponds the coupling strength between the circuits. The parameter α is the strength of negative resistor. The nonlinear function $f(y_n - z_n)$ corresponds to the characteristics of the nonlinear resistor consisting of the diodes and described as follows: where $f(y)$ is described as follows :

$$f(y_n - z_n) = \begin{cases} y_n - z_n - 1 & (y_n - z_n > 1) \\ 0 & (|y_n - z_n| \leq 1) \\ y_n - z_n + 1 & (y_n - z_n < -1). \end{cases} \quad (5)$$

This circuit generates asymmetric chaotic attractor in the parameter $\gamma = 0.5$ and symmetric chaotic attractor in the parameter $\gamma = 0.65$ as shown in Fig. 2.

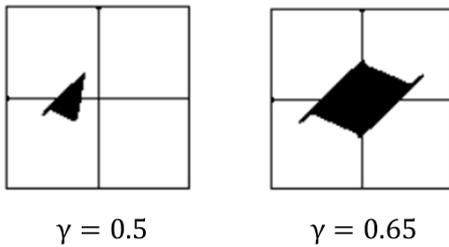


Fig. 2. Attractors with the different parameter.

III. NETWORK MODEL

We made the underlying network which is 10 coupled chaotic circuits and each circuit connected by a resistor. This network model and the degree distribution of this network are shown in Fig. 3.

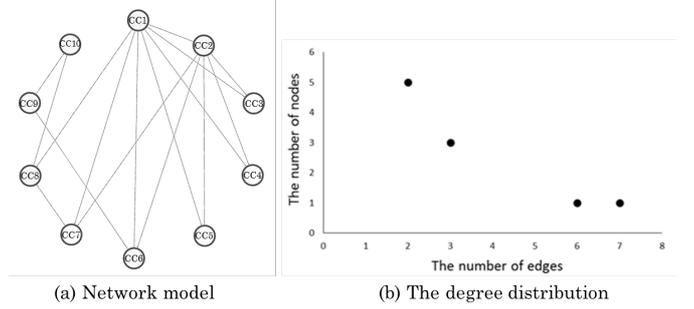


Fig. 3. A Underlying network model.

In this study, we propose network model which a hub node replaced with two distributed nodes. The example of splitting hub is shown in Fig. 4. Right network model in Fig. 5 is the network that applied method shown in Fig. 4 to CC1 as a hub of the Underlying network. Furthermore, we investigated synchronization of network in two different ways of connecting. In the first model, distributed hubs are connected by a positive resistor and other nodes are connected by a positive resistor each other. In the second model, distributed hubs are connected by a negative resistor and other nodes are same at the first model.

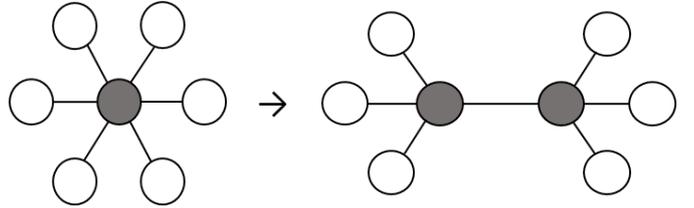


Fig. 4. Splitting a node.

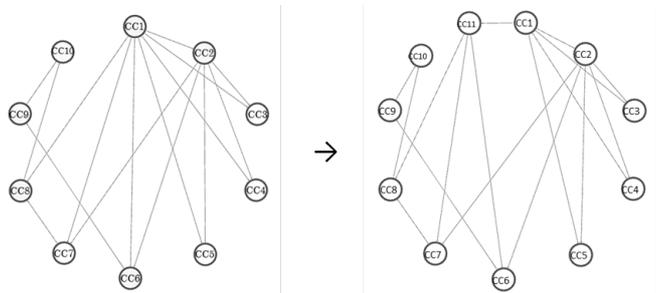


Fig. 5. Network model with distributed hub.

IV. SIMULATION RESULT

Definition of synchronization in this study is determined a voltage difference waveform. We define synchronization as the following Eq. (6).

$$|z_j - z_i| < 0.15 \quad (i, j = 1, 2, \dots, 10) \quad (6)$$

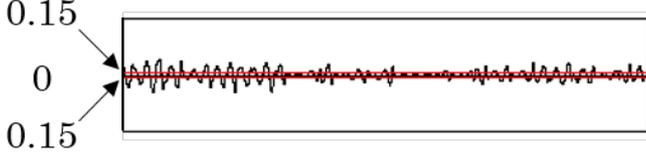


Fig. 6. A difference waveform.

Figure 6 is a different waveform which was observed in this simulation. The two lines in Fig. 6 correspond threshold which is given Eq. (6). It is determined that wave within two lines which is the threshold is synchronization. Therefore, we propose and investigate the synchronization probability denoted the synchronization rate during a certain time interval. In this research, we fix a certain time interval as ($\tau=1,000,000$ and $\text{step}=0.02\tau$) and investigate the synchronization rate in the entire network of 10 coupled chaotic circuits.

A. Comparing network with distributed hub and underlying network

In this simulation, we use two parameters of the strength of negative resistor γ . Attractors of these parameters are shown in Fig. 2. We compared synchronization rate of underlying network and network with distributed hub nodes which connected by positive resistor. Where the coupling strength other than between distributed nodes is defined as δ and the coupling strength between distributed nodes is defined as δ' . In underlying network, we measured synchronization rate by changing the coupling strength $\delta = 0.01$ to $\delta = 0.1$. In network with distributed hub, we measured synchronization rate by changing the coupling strength $\delta = 0.01$ to $\delta = 0.01$ and the coupling strength $\delta' = 0.02$ to $\delta' = 0.2$. Then we calculated difference of synchronization rate of underlying network and network with distributed hub nodes in each coupling strength δ . Figure 7 shows average of synchronization rate of the coupling strength δ' in two parameters $\gamma = 0.5$ and $\gamma = 0.65$. In Fig. 7, we can observed that synchronization rate is almost unchanged, even if a hub is replaced with two distributed hub nodes.

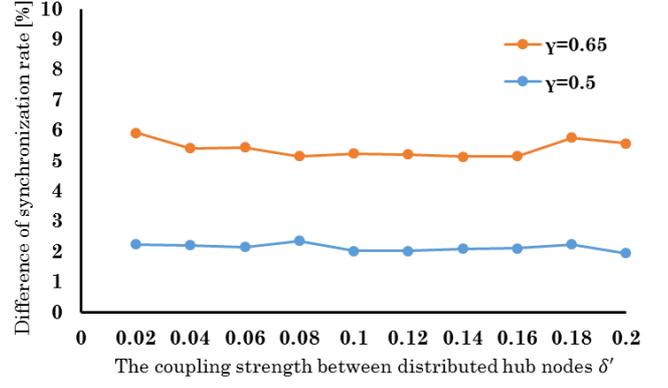


Fig. 7. Average of difference in synchronization rate.

B. Synchronization phenomena of network with distributed hub which connected by negative resistor

In this section, we observed synchronization phenomena of network with distributed hub nodes which connected by negative resistor. We measured synchronization rate the network by changing the coupling strength $\delta' = -0.02$ to $\delta' = -0.2$ in the coupling strength $\delta = 0.1$. Then, we grouped edges that are similar in connection and we averaged synchronization rate for each group. The first group is edges connected to CC1 which is distributed hub. The second group is edges connected CC11 which is distributed hub. The third group is edges that have a close relationship with node connected to CC1 of distributed hub. The fourth group is edges that have a close relationship with node connected to CC11 of distributed hub. The fifth group is edges between node connected to CC1 and node connected to CC11, that is, connection between nodes which are coupled with nodes that are in anti-phase relationship.

Figure 8 shows averages of synchronization rate of each group in the coupling strength $\delta' = -0.02$ to $\delta' = -0.2$. In Fig. 8, we can observed that synchronization rates of the first group and the second group are similar and that of the third group and the fourth group are similar. Synchronization rates of the first group and the second group are lower as the negative coupling is stronger. Synchronization rates of the third group and the fourth group are almost unchanged even if the negative coupling changed. Synchronization rates of the fifth group are lower as the negative coupling is stronger, although how to coupling of fifth group is similar to the third group and the fourth group. This results are considered that fifth group is connections between nodes which are coupled with nodes that are in anti-phase relationship.

V. CONCLUSIONS

In this study, we have investigated synchronization phenomena of network whose hub node is replaced with two distributed hub nodes. Further, we connected distributed hub nodes in two ways. In the first model, distributed hub nodes

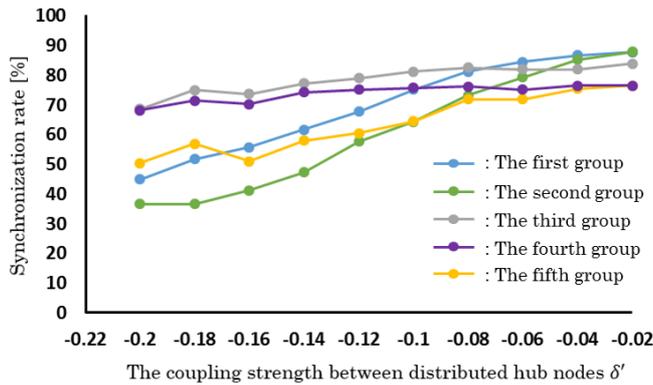
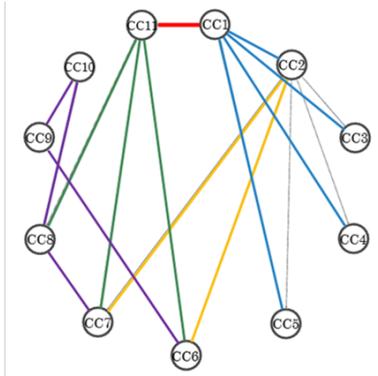


Fig. 8. Synchronization rate of each group.

connected by a positive resistor. In the second model, distributed hub nodes connected by a negative resistor.

In the result of the first model, when we compared synchronization rate of underlying network and network with distributed hub, we can observe that synchronization rate is almost unchanged, even if a hub is replaced with two distributed hub nodes. In the simulation of the second model, we compare edges grouping similar connection. From the simulation result, synchronization rates of group which is connection between nodes which are coupled with nodes that are in anti-phase relationship are lower as the negative coupling is stronger.

As our future work, we investigate network with distributed hub nodes coupled by negative resistor in detail. And we need to investigate synchronization phenomena of network with distributed hub nodes in larger network.

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