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Synchronization Phenomena in Weighted Complex Networks Using Chaotic Circuits

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***Abstract*—In this study, we investigate synchronization phenomena in weighted complex networks using chaotic circuit. The proposed chaotic circuits network has Scale-free distribution and the chaotic circuits are coupled by using distance information. Next, we focus on the role of hubs in the proposed networks. By computer simulations, we observe that the synchronization rate between hubs is high regardless of the coupling strength.**

I. Introduction

Many nonlinear phenomena in our society can be expressed using complex networks. For example, computer network, neuron, firefly synchronization. In such complex networks, synchronization phenomena are very important things.

In this study, we investigate synchronization phenomena in weighted complex networks using chaotic circuits. In this circuit system, the chaotic circuits are coupled with the distance information. Furthermore, we study influence of hub to synchronization in complex network. We use a chaotic circuit called "Mori-sinnriki Circuit" as a node. In this study, we use weighted complex networks with two hubs.

In this simulation, we investigate two experiments using this circuit. First, we focus on the distance between hubs. The nodes other than hubs which are connected with fewer nodes. We fix the position of hubs. Furthermore, we create another new hub by increasing links of another node. We investigate the synchronization rate between the fixed hub and the new hub.

Second, we focus on a node which connect to the hub. We build a network model with two hubs which are connecting to different nodes. In this network, we can confirm effect of a node which connect to hubs. And we investigate the synchronization rate between the hubs.

By computer simulations, we observe that the synchronization rate between hubs is high regardless of the coupling strength.

II. Circuit model

The chaotic circuit model which called Mori-Sinriki chaos circuit is shown in Fig. 1. This circuit consists of one negative resister, two capacitors, one inductors and dual-directional three diodes. And this circuit equation is shown Eq. (1).

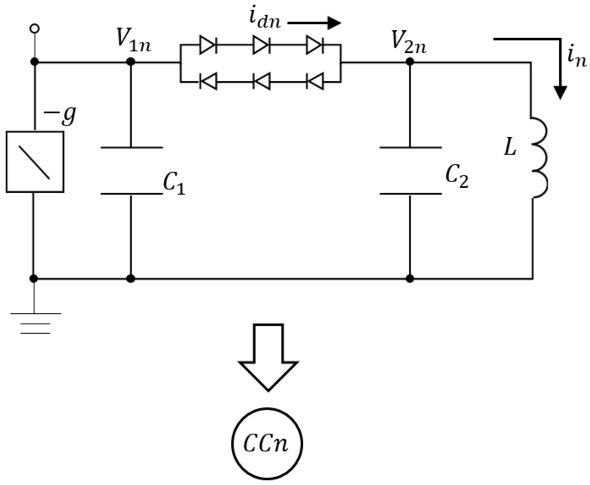


Fig. 1: Circuit model.

$$\begin{cases} L\dfrac{di_1}{dt} = V_{2n}, \\ C_1\dfrac{dV_{1n}}{dt} = gV_{1n} - i_{dn}, \\ C_2\dfrac{dV_{2n}}{dt} = i_{dn} - i_n. \end{cases} \quad (1)$$

The characteristic of nonlinear resistance which consists of dual three diodes is following Eq. (2).

$$i_{dn} = \begin{cases} Gd(V_{1n} - V_{2n} - V) \quad (V_{1n} - V_{2n} > V), \\ 0, \quad (|V_{1n} - V_{2n}| < V), \\ Gd(V_{1n} - V_{2n} + V), \quad (V_{1n} - V_{2n} < -V). \end{cases} \quad (2)$$

By changing the variables and parameters Eq. (3),

$$
\begin{cases}
i_n = \sqrt{\dfrac{C_2}{L}}Vx_n, \quad V_{1n} = Vy_n, \quad V_{2n} = Vz_n \\
t = \sqrt{LC_2}\tau, \quad \alpha = \dfrac{C_2}{C_1}, \\
\beta = \sqrt{\dfrac{L}{C_2}}Gd, \quad \gamma = \sqrt{\dfrac{L}{C_2}}g, \quad \delta = \dfrac{1}{R}\sqrt{\dfrac{L}{C_2}}.
\end{cases} \tag{3}
$$

The normalized equation of this circuit is given as follows:

$$
\begin{cases}
\dfrac{dx}{d\tau} = z_n, \\
\dfrac{dy}{d\tau} = \alpha\gamma y_n - \alpha f(y_n - z_n), \\
\dfrac{dz}{d\tau} = f(y_n - z_n) - x_n.
\end{cases} \tag{4}
$$

where $f(y_n - z_n)$ is described as follows :

$$
f(y_n - z_n) = \begin{cases}
\beta(y_n - z_n - 1) \quad (y_n - z_n > 1), \\
0, \quad (|y_n - z_n| < 1), \\
\beta(y_n - z_n + 1), \quad (y_n - z_n < -1).
\end{cases} \tag{5}
$$

III. System model

We investigate the synchronization rate using three networks based on original network. The original network model is shown in Fig. 2. The network is composed of 10 chaotic circuits and has Scale-free distribution. In this network, CC1 and CC2 are fixed hubs.

Figure 3 shows three networks obtained from the original network. First, Fig. 3 (a) shows the network that CC4 is added 4 links (connecting to CC3, CC5, CC6 and CC7). Namely, CC4 becomes new hub in this network. Similarly, Fig. 3 (b) shows the network that CC8 is added 4 links (connecting to CC2, CC3, CC4 and CC5). Namely, CC8 becomes new hub in this network. In Fig. 3 (c), two hubs connect to the different nodes. Also, the hub can connect only four nodes. If the original network remains, there are other hubs left behind, which may affect two hubs. Accordingly, we decrease other hub for focusing synchronization rate between hubs.

We compared the synchronization rate between hubs by changing the coupling strengths.

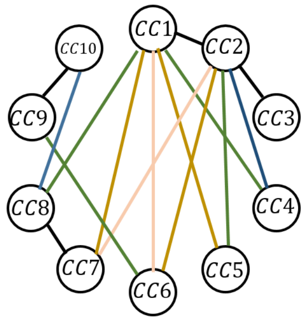
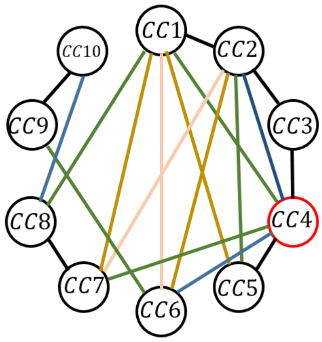
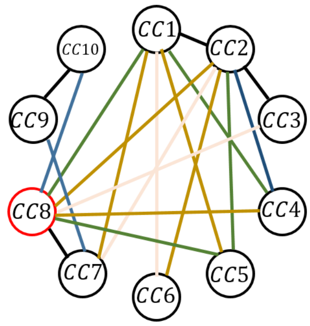


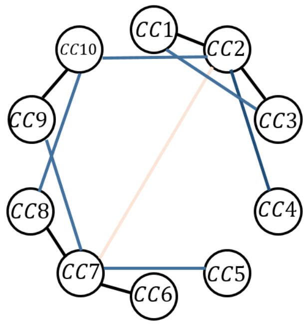
Fig. 2: Original network model.



(a) Network with new hub:CC4.



(b) Network with new hub:CC8.



(c) Network with divide links.

Fig. 3: Proposed network.

The normalized circuit equation of these network models is given by the following equations.

$$\begin{cases} \dfrac{dx}{d\tau} = z_n, \\ \dfrac{dy}{d\tau} = \alpha\gamma y_n - \alpha f(y_n - z_n) - \alpha\delta \displaystyle\sum_{k \in S_n} (y_n - y_k), \\ \dfrac{dz}{d\tau} = f(y_n - z_n) - x_n. \end{cases} \quad (6)$$

The parameter δ corresponds the coupling strength between the circuits. And we set the parameter α = 0.500, β = 20.000 and γ = 0.500.

The coupling strength of each link is determined by the distance. The coupling strength becomes weaker as the length becomes longer. In Fig. 4, there are five kinds of the coupling strength. We define shortest links as a base length (coupling strength define 1.000). The coupling strength of other links is defined by the reciprocal of the ratio of length to base. An accurate value of the five kinds of coupling strength is summarized in Table 1.

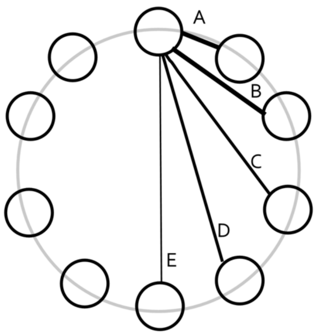


Fig. 4: Coupling strength.

TABLE I: Coupling strength.

	A	B	C	D	E
coupling strength	1.000	0.5257	0.3826	0.3249	0.3090

IV. Simulation results

Definition of synchronization in this study is determined a voltage waveform. We define synchronization as the following Eq. (7).

Figure 5 is a different wave form which was observed. Red line corresponds threshold which is given Eq. (7). It is determined that wave within red line which is the threshold is synchronization. Also, we set the synchronization rate by calculating the ratio at all measured points.

$$|y_j - y_i| < 0.03 \qquad (i, j = 1, 2, \cdots, 10) \qquad (7)$$

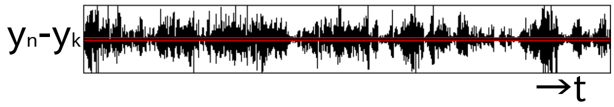


Fig. 5: Different wave form.

First, we consider a network model in Fig. 3 (a) and (b) and focus on CC1-CC2 to CC1-CC8. In Fig. 6, CC1-CC4 synchronization rate is higher than CC1-CC3 when spite of coupling strength is lower. Similarly, in Fig. 7, CC1-CC8 synchronization rate is higher than CC1-CC4. These two synchronization rate is increased from the original network model.

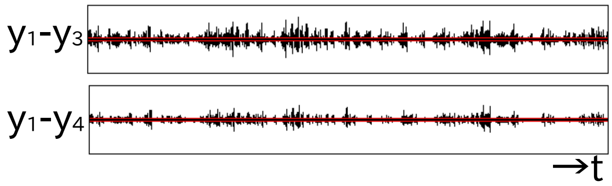


Fig. 6: Different wave form in Fig. 3 (a).

TABLE II: Synchronization rate of Fig. 6.

Links	CC1-CC3	CC1-CC4
Synchronization rate	32%	50%

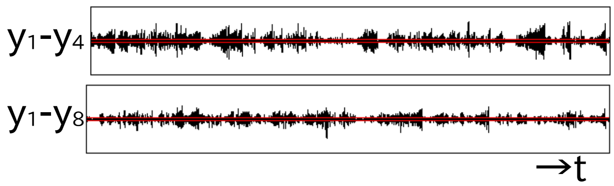


Fig. 7: Different wave form in Fig. 3 (b).

TABLE III: Synchronization rate of Fig. 7.

Links	CC1-CC4	CC1-CC8
Synchronization rate	26%	40%

We assume that synchronization rate between hubs is determined by not only the coupling strength unconditionally. We decrease the coupling strength between fixed and new hubs. In Fig. 10, we set the coupling strength of CC1-CC4 to 0. Similarly, in the Fig. 11, we set the coupling strength of CC1-CC8 to 0. Although the coupling strength was set to 0, the synchronization rate dose not change so much in Tables IV and V.

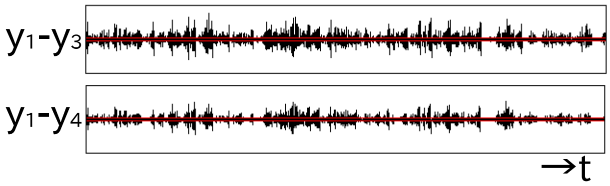


Fig. 8: Different wave form in Fig. 3 (a).

TABLE IV: Synchronization rate of Fig. 8.

Links	CC1-CC3	CC1-CC4
Synchronization rate	29%	38%

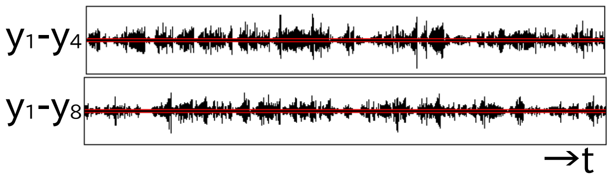


Fig. 9: Different wave form in Fig. 3 (b).

TABLE V: Synchronization rate of Fig. 9.

Links	CC1-CC4	CC1-CC8
Synchronization rate	24%	29%

Next, we split two hubs which links connect to divide nodes. In Fig. 3(c), we set CC2 and CC7 as the hubs. And we change the coupling strength between CC2-CC7. In Fig. 10, we compare in case of two coupling strength (δ=0.2 and 0.0). From this result, we can see that the synchronization rate between hubs has relationship with the connecting nodes.

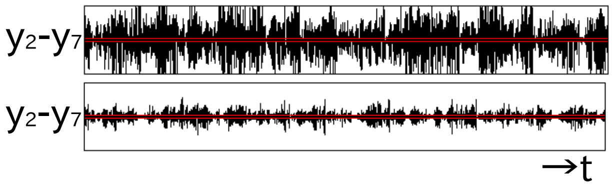


Fig. 10: Different wave form in Fig. 3 (c).

TABLE VI: Synchronization rate of Fig. 10.

Coupling strength of CC2-CC7	0	2.0
Synchronization rate	26%	6%

V. Conclusions

In this study, we have investigated the influence of hub by using various networks changing hubs position. Also, we investigate about reasons which synchronization rate between hubs is higher in two methods. First, we change the coupling strength between hubs. Next, we split two hubs by divide connecting nodes.

In this result, the synchronization rate between the hubs is high regardless the coupling strength. We tried to investigate reasons of high synchronization rate. As a result, in the case of between hubs not connected to the same node, synchronization rate is proportional the coupling strength. Furthermore, this result shows reason two synchronization rate between hubs are differences in same coupling strength. We observe that it occurs because the strongly connected nodes were different. We seem the coupling strength between hubs involved to the connected nodes.

In our study, we use only five coupling strength value depending on the distance between chaotic circuits. Also, a network which used in this study is very small (N=10). We need to use another type network and would like to investigate the synchronization rate by using the same methods. Furthermore, so we should use bigger networks inspired from real biological networks.

References

[1] K. Ago, Y. Uwate and Y. Nishio “Synchronization of Coupled Chaotic Circuits with Parameter Dispersion in Small-World Network”,Proceedings of International Symposium on Nonlinear Theory and its Applications,pp. 431-434,Dec. 2015.

[2] H. Shuhei, T. Chikazawa, Y. Uwate and Y. Nishio, “Synchronization Phenomena of Complex Networks in Coupled Chaotic Circuits with Different Degree Distribution”,Proceedings of RISP International Workshop on Nonlinear Circuits, Communications and Signal Processing (NCSP’17), pp. 333-336, Feb. 2017.