1-19

Effect of Changing Oscillator Frequencies to Synchronization Phenomena in Coupled van der Pol Oscillators

Vu Minh HIEN　Minh Hai TRAN　Yoko UWATE　Yoshifumi NISHIO
(Tokushima University)

1. Introduction

In this study, we use van der Pol oscillators coupled by one resistor when the oscillator frequencies are changed. By using computer simulations, we can observe the effect of the changing to synchronization phenomena in these coupled oscillators.

2. System Model

Figure 1 shows a system model consisting of van der Pol oscillators.

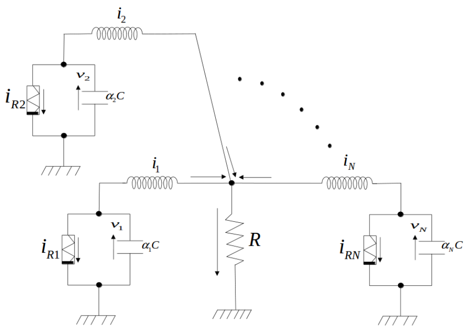


Figure 1: Circuit model.

The circuit equations are described as follows:

$$\begin{cases} \alpha_k C \dfrac{dv_k}{dt} = -i_k + g_1 v_k - g_3 v_k^3 \\ L\dfrac{di_k}{dt} = -v_k - R\sum_{j=1}^{N} i_j \end{cases} \quad (1)$$

$$(k = 1, 2, 3, ..., N)$$

To normalize circuit equations, we use Eq. (2) to change the variables and the parameters:

$$\begin{cases} t = \sqrt{LC}\tau \\ v_k = \sqrt{\dfrac{g_1}{3g_3}} x_k, \quad i_k = \sqrt{\dfrac{Cg_1}{3Lg_3}} y_k \\ \alpha_k = \dfrac{1}{\omega_k^2}, \quad \beta = g_1\sqrt{\dfrac{C}{L}}, \quad \varepsilon = R\sqrt{\dfrac{C}{L}} \end{cases} \quad (2)$$

Eq. (1) is normalized as:

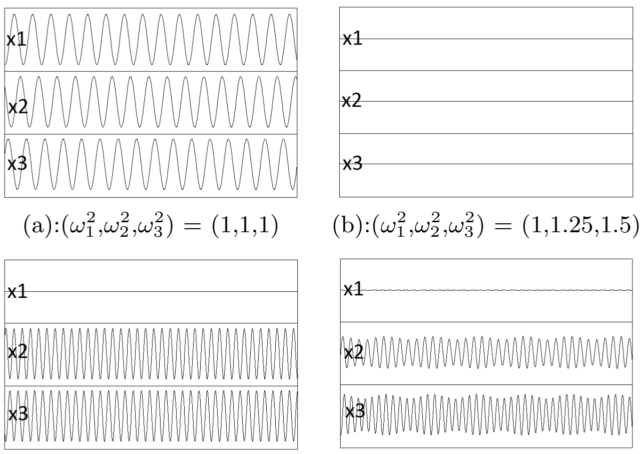
$$\begin{cases} \dfrac{dx_k}{dt} = \omega_k^2 \left\{ \varepsilon \left(x_k - \dfrac{1}{3}x_k^3 \right) - y_k \right\} \\ \dfrac{dy_k}{dt} = x_k - \beta \sum_{j=1}^{N} y_j \end{cases} \quad (3)$$

where ε is the coupling factor, β is the strength of nonlinearity, and α_k is a factor that added to each capacitor. In this article, the number of oscillator is set to three. We change the frequencies of three oscillators.

3. Simulation results

Figure 2 shows the observed phenomana for $N = 3$. In the case of $(\omega_1^2,\omega_2^2,\omega_3^2) = (1,1,1)$, the all three oscillators oscillated as Fig. 2(a). But when we slightly change the frequencies of the $2nd$ and the $3rd$ oscillators, all of three oscillators stop, namely oscillation death appears as Fig. 2(b) where $(\omega_1^2,\omega_2^2,\omega_3^2) = (1,1.25,1.5)$.

And when we continue to increase the frequencies of the $2nd$ and the $3rd$ oscillators with the same values $(\omega_1^2,\omega_2^2,\omega_3^2) = (1,5,5)$, the $2nd$ and the $3rd$ oscillators work again as Fig. 2(c). Then we change to different frequencies, the amplitudes of the $2nd$ and the $3rd$ oscillators start to change as beat signals. Figure 2(d) shows the result when $(\omega_1^2,\omega_2^2,\omega_3^2) = (1,5,7)$. In the case of changing to diferrent frequencies of all three oscillator, we also obtain the same result. Therefore, the different frequencies effected to amplitudes of the coupled oscillators.



(a):$(\omega_1^2,\omega_2^2,\omega_3^2) = (1,1,1)$　(b):$(\omega_1^2,\omega_2^2,\omega_3^2) = (1,1.25,1.5)$

(c):$(\omega_1^2,\omega_2^2,\omega_3^2) = (1,5,5)$　(d):$(\omega_1^2,\omega_2^2,\omega_3^2) = (1,5,7)$

Figure 2: Simulation results for $N = 3$.

4. Conclusion

In this study, we have investigated the effect of changing oscillator frequencies to synchronization phenomena. By using computer simulations, we confirmed that oscillation of the oscillators stop, namely oscillation death in some range of the frequencies. By increasing the diferrent frequencies, oscillators not only work again but also the amplitudes start to change as beat signals.

In the future, we increase the number of the oscillators and use theoretical analysis to comfirm these results.