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Synchronization of Coupled Six van der Pol Oscillators Containing Two Rings

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1. Introduction

In this study, we propose a novel coupled oscillatory system. By computer simulations and circuit experiments, we investigate synchronization phenomena observed in the proposed circuit system by changing one of the coupling strength.

2. System model

Figure 1 shows two circuits and a model of the system. The circuit of a van der Pol oscillator is called circuit VDP. The circuit with a slightly modified VDP structure is called NC. We use six van der Pol oscillators in this study. We use two ring circuits of van der Pol oscillators. The three VDP of the first ring are connected by resistors. The three NC of the second ring are connected by inductors and resistors. When the two rings are not connected, the oscillators in the first ring exhibit in-phase synchronization and oscillators of the second ring exhibit three-phase synchronization. The first and second rings are connected by resistors (R_1, R_2, R_3). We observe the synchronization phenomena of adjacent oscillators. We investigate how the synchronization phenomena change upon changing the value of R_2.

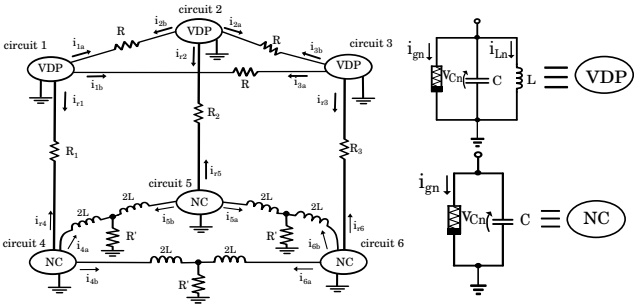

Figure 1: Circuit model.

The normalized equations of the first ring are given as:

$$\begin{cases} \dot{x}_n &= \varepsilon(x_n - x_n^3) - y_n - \gamma_n(x_n - x_{n+3}) \\ & \quad +\alpha(-x_n + x_i + x_j) \\ \dot{y}_n &= x_n \end{cases} \qquad (n = 1, 2, 3) \tag{1}$$

and the normalized equations of the second ring are given as:

$$\begin{cases} \dot{x}_n &= \varepsilon(x_n - x_n^3) - y_{an} \\ & \quad -y_{bn} + \gamma_{n-3}(x_n - x_{n-3}) \\ \dot{y}_{an} &= x_n - \beta(y_{an} + y_{b(i)}) \\ \dot{y}_{bn} &= x_n - \beta(y_{bn} + y_{a(j)}) \end{cases} \qquad (n = 4, 5, 6) \tag{2}$$

where n, i and j is denotes the number of the circuit ($n = 1, 2, 3, 4, 5, 6$, $i = 2, 3, 1, 5, 6, 4$ and $j = 3, 1, 2, 6, 4, 5$.) The parameters ε is non-linear strength. The parameters α, β and γ denote the coupling strengths of the resistor R, resistor R' and resistor R_n, respectively.

3. Results

The simulation results of the system model are shown in Fig. 2 and 3. The parameters are set to $\varepsilon = 0.05$, $\alpha = 0.05$ and $\beta = 0.05$. Figure 2 show the graph of the relationship between coupling strength and phase difference. We change the value of γ_2 from 0 to 0.03 at intervals of 0.001. We fix the value of other coupling strengths and initial value. When γ_2 is even smaller than 0.020, circuit 4 - circuit 6 exhibit in-phase synchronization.

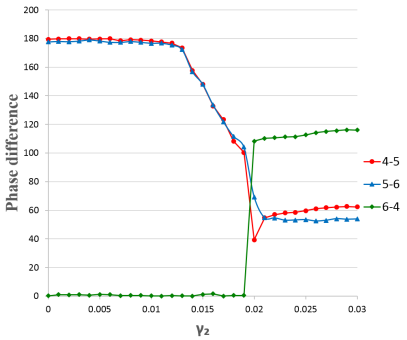

Figure 2: The graph of phase difference.

In the case $\gamma_1 = \gamma_3 = 0.02$ and $\gamma_2 = 0.03$, Fig. 3 show the time waveform of the first ring and the second ring. Oscillators of first ring exhibit in-phase synchronization and oscillators of second ring exhibit synchronization that three-phase synchronization gravitate toward in-phase synchronization. The time waveform of circuit experiments is shown in Fig. 4. The element value are set to $L = 10$mH, $C = 22$nF, $R' = 1.2$kΩ, $R_1 = R_3 = 1.56$kΩ and $R_2 = 560\Omega$. We obtained results of circuit experiments similar to simulation results.

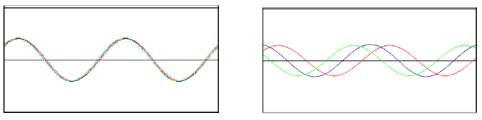

Figure 3: The time waveform (computer simulations).

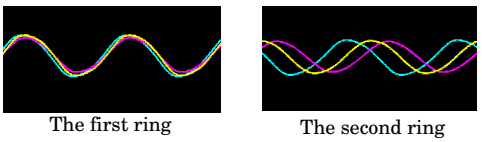

Figure 4: The time waveform (circuit experiments).

4. Conclusions

We have proposed a system model using two rings of three van der Pol oscillators coupled by resistors or inductors. We investigated synchronization how the synchronization phenomena change upon changing the value of γ_2. In addition, we obtained results of circuit experiment similar to simulation results. In the future, we will investigate synchronization phenomena using other parameters and analyze the proposed circuit model.