Synchronization Phenomena in a Ring of van der Pol Oscillators Coupled by Time-Varying Resistor

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1. Introduction

Synchronization is one of the fundamental phenomena in nature and it is observed over the various fields.

In this study, we propose coupled oscillatory systems such as a ring of van der Pol oscillators coupled by Time-Varying Resistor (TVR). We investigate synchronization phenomena observed in the proposed circuit system by changing the frequency of TVR.

2. System model

Figure 1 shows a system model constituted van der Pol oscillators (VDP). We use a ring of van der Pol oscillators, three VDP are connected by Time-Varying Resistor (TVR1 and TVR2). We realize the TVR by switching a positive and a negative resistor periodically as shown in Fig. 2. We investigate synchronization phenomena, how changing by the frequency of TVR1 and TVR2.

The normalized equations of VDP circuit are given as follows:

\[
\begin{align*}
\dot{x}_1 &= \varepsilon(x_1 - x_1^3) - y_1 + \gamma_1(x_2 + x_3 - 2x_1) \\
\dot{x}_2 &= \varepsilon(x_2 - x_2^3) - y_2 + \gamma_1(x_1 - x_2) + \gamma_2(x_3 - x_2) \\
\dot{x}_3 &= \varepsilon(x_3 - x_3^3) - y_3 + \gamma_1(x_1 - x_3) + \gamma_2(x_2 - x_3) \\
y_n &= x_n.
\end{align*}
\]  

(1)

In these equations, \( n \) is the number of circuit and \( n = 1,2,3 \). \( \varepsilon \) denotes the nonlinearity of the oscillators, \( \gamma_1 \), and \( \gamma_2 \) denote the coupling strengths of the TVR1 and TVR2.

3. Simulation results

The simulation results of the system model are shown from Fig. 3 to Fig. 6. The value of the parameters are set to \( \varepsilon = 0.1, \gamma_1 = 0.01, \gamma_2 = 0.01 \). A frequency of TVR1 and TVR2 sets with \( f_1 \) and \( f_2 \). Where, \( f_n = 2\pi\omega_n \). The figure on the left shows phase difference when the initial condition set with in-phase. The figure on the right shows phase difference when the initial condition set with 3-phase. In case of \( f_1 = f_2 = 0.09 \), we can observe in-phase synchronization phenomena and 3-phase synchronization phenomena. We can observe both synchronization phenomena due to change by initial value. By changing \( f_1 \) and \( f_2 \), we observe only 3-phase synchronization phenomena, it is observed regardless of initial value. In case of \( f_1 = 0.09, f_2 = 0.1 \), we can observe both synchronization phenomena. In case of \( f_1 = 0.03, f_2 = 0.219 \), circuit2 and circuit3 exhibit synchronization phenomena regardless of initial value. We observe synchronization phenomena by changing the frequency.

4. Conclusions

We have proposed a system model using a ring of three van der Pol oscillators coupled by TVR. We can observe various synchronization phenomena by varying frequency of TVR. When two frequencies of TVR \( (f_1, f_2) \) equal, sometimes we can observe synchronization phenomena, and sometimes 3-phase synchronization phenomena. However when we changed frequency \( f_1 \) and \( f_2 \), three oscillators of a ring possible to see behavior different from equivalent in a frequency. In the future, we will investigate the synchronization phenomena using different frequency of the three TVRs.