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Synchronization Phenomena in a Ring of van der Pol Oscillators Coupled by Time-Varying Resistor

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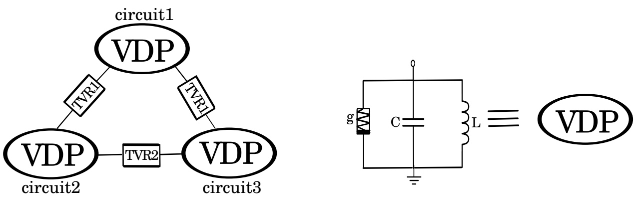
1. Introduction

Synchronization is one of the fundamental phenomena in nature and it is observed over the various fields.

In this study, we propose coupled oscillatory systems such as a ring of van der Pol oscillators coupled by Time-Varying Resistor (TVR). We investigate synchronization phenomena observed in the proposed circuit system by changing the frequency of TVR.

2. System model

Figure 1 shows a system model constituted van der Pol oscillators (VDP). We use a ring of van der Pol oscillators, three VDP are connected by Time-Varying Resistor (TVR1 and TVR2). We realize the TVR by switching a positive and a negative resistor periodically as shown in Fig. 2. We investigate synchronization phenomena, how changing by the frequency of TVR1 and TVR2.



(a) System model. (b) van der Pol oscillator.

Figure 1: Circuit model.

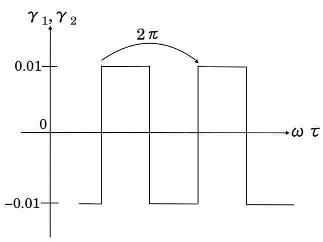


Figure 2: Characteristics of the TVR.

The normalized equations of VDP circuit are given as follows:

$$
\left\{
\begin{array}{rcl}
\dot{x_1} &=& \varepsilon(x_1 - x_1^3) - y_1 + \gamma_1(x_2 + x_3 - 2x_1) \\
\dot{x_2} &=& \varepsilon(x_2 - x_2^3) - y_2 + \gamma_1(x_1 - x_2) + \gamma_2(x_3 - x_2) \\
\dot{x_3} &=& \varepsilon(x_3 - x_3^3) - y_3 + \gamma_1(x_1 - x_3) + \gamma_2(x_2 - x_3) \\
\dot{y_n} &=& x_n.
\end{array}
\right. \quad (1)
$$

In these equations, n is the number of circuit and n = 1,2,3. ε denotes the nonlinearity of the oscillators, γ_1, and γ_2 denote the coupling strengths of the TVR1 and TVR2.

3. Simulation results

The simulation results of the system model are shown from Fig. 3 to Fig. 6. The value of the parameters are set to $\varepsilon = 0.1$, $\gamma_1 = \pm 0.01$, $\gamma_2 = \pm 0.01$. A frequency of TVR1 and TVR2 sets with f_1 and f_2. Where, $f_n = 2\pi\omega_n$. The figure on the left shows phase difference when the initial condition set with in-phase. The figure on the right shows phase difference when the initial condition set with 3-phase. In case of $f_1 = f_2 = 0.09$, we can observe in-phase synchronization phenomena and 3-phase synchronization phenomena. We can observe both synchronization phenomena due to change by initial value. By chaging f_1 and f_2, we observe only 3-phase synchronization phenomena, it is observed regardless of initial value. In case of $f_1 = 0.09$, $f_2 = 0.1$, we can observe both synchronization phenomena. In case of $f_1 = 0.03$, $f_2 = 0.219$, circuit2 and circuit3 exhibit synchronization phenomena regardless of initial value. We observe synchronization phenomena by changing the frequency.

(a) in-phase (b) 3-phase

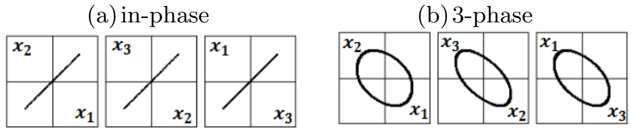


Figure 3: Phase difference ($f_1 = f_2 = 0.09$).

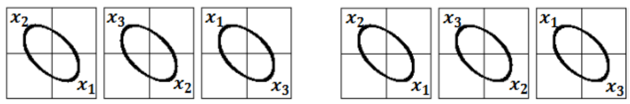


Figure 4: Phase difference ($f_1 = f_2 = 0.1$).

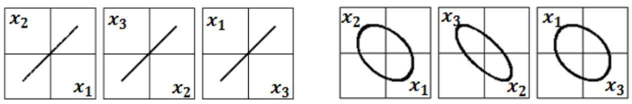


Figure 5: Phase difference ($f_1 = 0.09$, $f_2 = 0.1$).

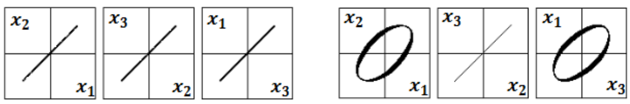


Figure 6: Phase difference ($f_1 = 0.03$, $f_2 = 0.219$).

4. Conclusions

We have proposed a system model using a ring of three van der Pol oscillators coupled by TVR. We can observe various synchronization phenomena by varying frequency of TVR. When two frequencies of TVR (f_1, f_2) equal, sometimes we can observe synchronization phenomena, and sometimes 3-phase synchronization phenomena. However when we changed frequency f_1 and f_2, three oscillators of a ring possible to see behavior different from equivalent in a frequency. In the future, we will investigate the synchronization phenomena using different frequency of the three TVRs.