Synchronization in Two Rings of Coupled van der Pol Oscillators

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I. Introduction

In this study, we propose a novel coupled oscillatory system such as two rings of van der Pol oscillators coupled by resistors. we investigate synchronization phenomena observed in the proposed circuit system by changing the coupling strength.

II. System model

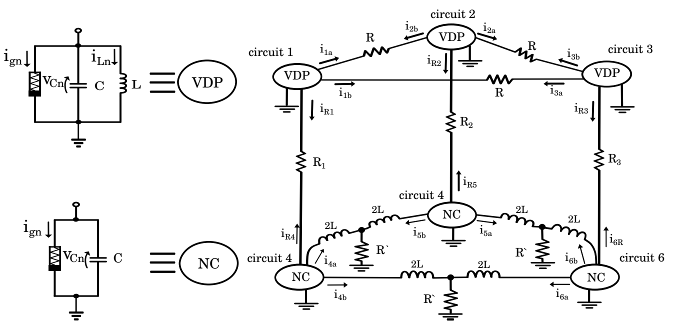

Fig. 1. Circuit model.

Figure 1 shows the circuit model which is used in my research. we use two ring circuits of six oscillators. Three VDP of the first ring are connected by resistors, three NC of the second ring are connected by inductors and resistors. The first and the second rings are connected by resistors (R_1, R_2, R_3). we investigate how to change synchronization phenomena of adjacent oscillators by changing the value of R_1, R_2 and R_3.

Nonlinear resistor is defined as follows:

$$i_{gn} = -g_1 v_n + g_3 v_n^3. \quad (1)$$

The normalized circuit equations of the first ring are given as follows:

$$\begin{cases} \dot{x}_n = \varepsilon(x_n - x_n^3) - y_n - \gamma(x_n - x_{n+3}) \\ \qquad +\alpha(-x_n + x_i + x_j) \\ \dot{y}_n = x_n. \end{cases} \quad (2)$$

The normalized circuit equations of the second ring are given as follows:

$$\begin{cases} \dot{x}_n = \varepsilon(x_n - x_n^3) - y_{an} \\ \qquad -y_{bn} + \gamma(x_n - x_{n-3}) \\ \dot{y}_{an} = x_n - \beta(y_{an} + y_{b(i)}) \\ \dot{y}_{bn} = x_n - \beta(y_{bn} + y_{a(j)}) \end{cases} \quad (3)$$

where $n = 1, 2, 3, 4, 5, 6$. The parameters ε, α, β, and γ denote the coupling strengh of the inductor, resistor R, resistor R', and resistor R_n, respectively.

III. Simulation results

The simulation results of the system model are shown from Fig. 2 to Fig. 5. The value of the parameters are set to $\varepsilon = 0.05$, $\alpha = 0.05$, $\beta = 0.05$. In the case of $\gamma_1 = \gamma_2 = \gamma_3 = 0.02$, in the second ring, synchronization phenomena change by initial value. By chaging γ_1, γ_2 and γ_3, we can control synchronization phenomena regardless of initial value. In the case of $\gamma_1 = 0.02, \gamma_2 = 0.005$ and $\gamma_3 = 0.02$, circuit 4 and circuit 6 become synchronized without reference to initial value. In the case of $\gamma_1 = 0.001, \gamma_2 = 0.0001$ and $\gamma_3 = 0.02$, the oscillators of the first ring become synchronized, the oscillators of the second ring become 3-phase synchronization. we observe synchronization phenomena by changing the coupling strengths.

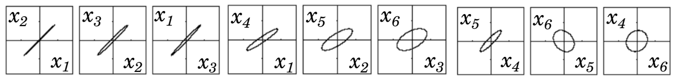

Fig. 2. Phase difference ($\gamma_1 = \gamma_2 = \gamma_3 = 0.02$)

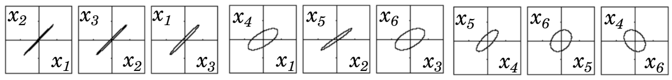

Fig. 3. Phase difference ($\gamma_1 = \gamma_2 = \gamma_3 = 0.02$)

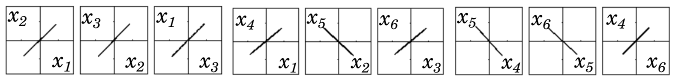

Fig. 4. Phase difference ($\gamma_1 = 0.001, \gamma_2 = 0.001, \gamma_3 = 0.02$)

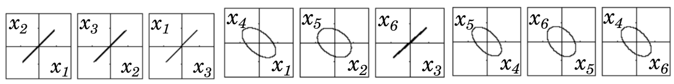

Fig. 5. Phase difference ($\gamma_1 = 0.02, \gamma_2 = 0.005, \gamma_3 = 0.02$)

IV. Conclusions

we have proposed a system model using two rings of three van der Pol oscillators coupled by resistors or inductors. we can control the synchronization phenomena by varying the coupling strengths.

References

[1] Y. Uwate, Y. Nishio, "Two van der Pol Oscillators Coupled by Chaotically Varying Resistor" Proceedings of International Workshop on Nonlinear Dynamics of Electronic Systems (NDES'06), pp. 189-192, Jun. 2006.