Synchronization in Two Rings of Coupled van der Pol Oscillators

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I. INTRODUCTION
In this study, we propose a novel coupled oscillatory system such as two rings of van der Pol oscillators coupled by resistors. We investigate synchronization phenomena observed in the proposed circuit system by changing the coupling strength.

II. SYSTEM MODEL
The normalized circuit equations of the first ring are given as follows:

\[
\begin{align*}
\dot{x}_n &= \varepsilon (x_n - x_n^3) - y_n - \gamma (x_n - x_{n+3}) + \alpha (-x_n + x_i + x_j) \\
\dot{y}_n &= x_n.
\end{align*}
\]

The normalized circuit equations of the second ring are given as follows:

\[
\begin{align*}
\dot{x}_n &= \varepsilon (x_n - x_n^3) - y_{an} - \gamma (x_n - x_{n-3}) \\
\dot{y}_{an} &= x_n - \beta (y_{an} + y_{b(n)}) \\
\dot{y}_{bn} &= x_n - \beta (y_{bn} + y_{a(j)})
\end{align*}
\]

where \( n = 1, 2, 3, 4, 5, 6 \). The parameters \( \varepsilon, \alpha, \beta, \) and \( \gamma \) denote the coupling strength of the inductor, resistor \( R \), resistor \( R' \), and resistor \( R_n \), respectively.

III. SIMULATION RESULTS
The simulation results of the system model are shown from Fig. 2 to Fig. 5. The value of the parameters are set to \( \varepsilon = 0.05, \alpha = 0.05, \beta = 0.05 \). In the case of \( \gamma_1 = \gamma_2 = \gamma_3 = 0.02 \), in the second ring, synchronization phenomena change by initial value. By changing \( \gamma_1, \gamma_2 \) and \( \gamma_3 \), we can control synchronization phenomena regardless of initial value. In the case of \( \gamma_1 = 0.02, \gamma_2 = 0.005 \) and \( \gamma_3 = 0.02 \), circuit 4 and circuit 6 become synchronized without reference to initial value. In the case of \( \gamma_1 = 0.001, \gamma_2 = 0.0001 \) and \( \gamma_3 = 0.02 \), the oscillators of the first ring become synchronized, the oscillators of the second ring become 3-phase synchronization, we observe synchronization phenomena by changing the coupling strengths.

IV. CONCLUSIONS
We have proposed a system model using two rings of three van der Pol oscillators coupled by resistors or inductors. We can control the synchronization phenomena by varying the coupling strengths.

REFERENCES