Influence of Regional Change in Synchronization of Complex Networks in Coupled Parametrically Excited Oscillators

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***Abstract*— In this study, we investigate synchronization in complex network with two hubs by using parametrically excited van der Pol oscillators with parameter mismatch. By means of computer simulation, we confirm various change of synchronization probability in complex network, and observe effects on synchronization probability by changing structural metrics (degree and path length) in a subset of the nodes with larger mismatch in complex networks.**

I. Introduction

Synchronization is one of the fundamental phenomena in nature and it is observed over the various fields. Studies on synchronization phenomena of coupled oscillators are extensively carried out in various fields, physics [1], biology [2], engineering and so on. Synchronization generated in the system can model certain synchronization of natural rhythm phenomena. Parametric excitation circuit is one of resonant circuits, and it is important to investigate various nonlinear phenomena of the parametric excitation circuits for future engineering applications. In simple oscillator including parametric excitation, Ref. [3] reports that the almost periodic oscillation occurs if nonlinear inductor has saturation characteristic. Additionally the occurrence of chaos is referenced in Refs. [4] and [5].

In our research group, we have investigated synchronization of parametrically excited van der Pol oscillators [6]. By carrying out computer calculations for two or three subcircuits cases, we have confirmed that various kinds of synchronization phenomena of chaos are observed. In the case of two subcircuits, the anti-phase synchronization is observed. In the case of three subcircuits, self-switching phenomenon of synchronization states is observed.

However, we have investigated the only simple network models. It is important to investigate more complex network for the broad-ranging future engineering applications. In our previous study, we have challenged to investigate the synchronization and clustering in more complex network modified from “Dolphin social network” [7] by using parametrically excited van der Pol oscillators with small mismatch [8]. We have confirmed that the network with hubs can induce synchronization. Though, we have only investigated the effects on synchronization offered by the location and number of pieces of the hubs, and we have only added small mismatch (dispersion) in whole.

In this study, in order to investigate the influence of regional change in synchronization of complex network, we focus on relationship between the structural metrics in a subset of the nodes is included larger mismatch and synchronization in complex network. In order to research this relationship, we investigate synchronization in complex network by changing structural metrics (degree and path length) corresponding to a subset of the nodes is added larger mismatch than other nodes. First, we investigate the synchronization probability in the network by changing the degree of the three nodes added larger mismatch. Next, we investigate the synchronization probability in the network by changing the path length among hubs, a number of nodes with larger mismatch[1].

II. Proposed network model

Topological structures in complex networks can be evaluated by the typical structural metrics (degree, clustering coefficient and path length). First, degree is the number of edges on a node. Second, clustering coefficient is the number of actual links between neighbors of a node divided by the number of possible links between those neighbors. Third, path length is the shortest path in the network between two nodes.

In this research, we consider coupled parametrically excited oscillators complex network in Fig. 1. In this system, parametrically excited van der Pol oscillators are coupled by one resistor R. In this network, the number of nodes which is parametrically excited van der Pol oscillator is 62 and the number of edges which is one resister is 100. The 15th and 58th nodes are hubs in this network. The bond number of 58th node is 12, and the bond number of 15th node is 13.Table 1 shows the properties of the proposed network as shown in Fig. 1.

[1]This research is obtained by adding more particular investigation toward path length in the paper is accepted in IEEE Asia Pacific Conference on Circuits and Systems (APCCAS’16)

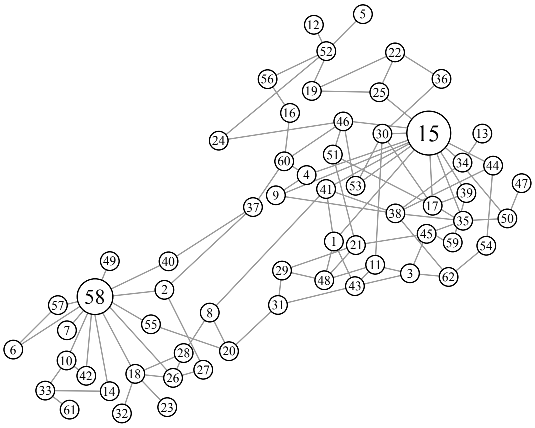


Fig. 1. Proposed network.

TABLE I
PROPERTIES OF PROPOSED NETWORK AS SHOWN IN FIG. 1.

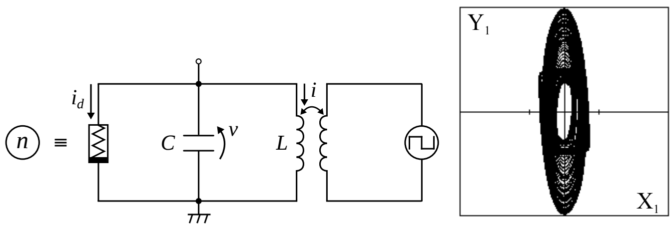
Average degree	3.226
Average clustering coefficient	0.145
Average path length	4.357

III. SYSTEM MODEL

The circuit model of van der Pol oscillator under parametric excitation is shown in Fig. 2 (a). The circuit includes a time-varying inductor L whose characteristics are given as the following equation.

$$L = L_0 \gamma(\tau). \tag{1}$$

The chaotic oscillation is expressed in Fig. 2 (b). $\gamma(\tau)$ is expressed in a rectangular wave as shown in Fig. 3, and its amplitude and angular frequency are termed α and ω, respectively. By changing the value of α, the amplitude of parametric excitation can be changed.



(a) Parametrically excited van der Pol oscillator.

(b) Chaotic oscillation ($\alpha = 0.950$).

Fig. 2. Circuit model.

The $v - i$ characteristics of the nonlinear resistor are approximated by the following equation.

$$i_d = -g_1 v_k + g_3 v_k. \tag{2}$$

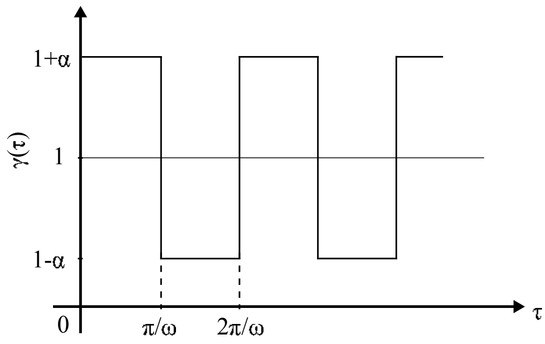


Fig. 3. Function relating to parametric excitation.

By changing the variables and the parameters,

$$\begin{cases} t = \sqrt{L_0 C}\tau, \ v_n = \sqrt{\dfrac{g_1}{g_3}} x_n, \ \omega = \omega_0 \sqrt{L_0 C} \\ i_n = \sqrt{\dfrac{g_1}{g_3}} \sqrt{\dfrac{C}{L_0}} y_n, \ \epsilon = g_1 \sqrt{\dfrac{L_0}{C}}, \ \delta = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}, \end{cases} \tag{3}$$

the normalized circuit equations are given by the following equations.

$$\begin{cases} \dfrac{dx_n}{d\tau} = \varepsilon(x_n - x_n^3) - y_n - \delta \displaystyle\sum_{k \in S_n} (x_n - x_k) \\ \dfrac{dy_n}{d\tau} = \dfrac{1}{\gamma(\tau)} x_n, \end{cases} \tag{4}$$

where $n = 1, 2, 3, ..., 62$. S_n is the set of nodes which are directly connected to the node n.

IV. INVESTIGATION OF THE SYNCHRONIZATION PROBABILITY

A. Synchronization probability

In this research, we investigate the synchronization by means of computer simulation. We calculate x_i by using Runge-Kutta method. Simulation time is $\tau = 4000\pi$ with a step size 0.002π. We fix the circuit parameters as $\varepsilon = 1.00$ and $\omega = 1.00$ for all circuits. Each circuit is given different initial values for computer simulations. In this simulation, all of the nodes involve small mismatch m_n in α (in Fig. 2) which is corresponding to the amplitude of the function relating to parametric excitation within the compass of $-0.001 \leq m_n \leq 0.001$.

Additionally, in order to analyze synchronous state, we define the synchronization by the following equation.

$$|x_n - x_k| < 0.10 \quad (k \in S_n). \tag{5}$$

The synchronization probability is the rate of the synchronized edges among the number of all of the edges (100). When the two oscillators (nodes) at the ends of an edge become synchronous state, we define this edge as synchronized edge.

B. The results of simulation

In these simulations, first, we fixed the coupling strength δ in 1.70 for this network is synchronized as completely. Next, we investigate change of the synchronization probability by adding larger mismatch M in a number of nodes. These nodes have a certain characteristic corresponding to the structural metrics. In order to investigate the relationship between the structural metrics in a subset of the nodes is included larger mismatch M and synchronization in complex network, we simulate the passage of the synchronization by adding and changing M from 0.002 to 0,01. This result can be shown in Fig. 5 : Fig. 9.

1) Effect of the degree including larger mismatch: In this simulation, we investigate change of synchronous state by adding larger mismatch M from 0.002 to 0.01 in 5 types node. These nodes have the degrees which is 1 to 5. We add the M in the amplitude of the function relating to parametric excitation $(1 + \alpha + m_n + M)$ in three nodes with the 5 types degrees (1 to 5). Figure 4 shows degree distribution of the proposed network. In this figure, vertical axis denotes the number of nodes, and horizontal axis denotes the value of degree.

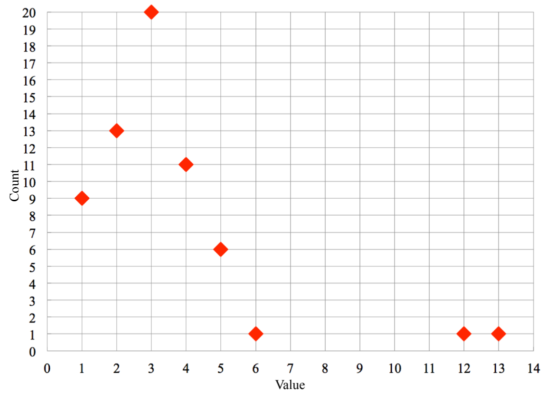
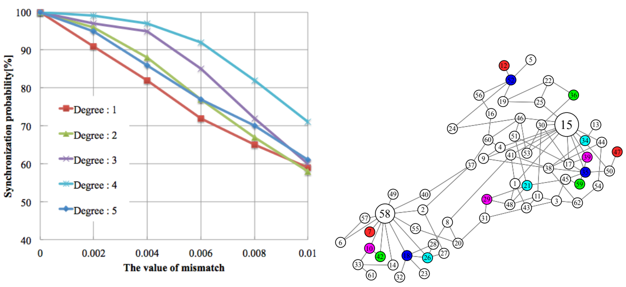


Fig. 4. Degree distribution of the proposed network.



(a) Synchronization probability.

(b) The location of the nodes with M.

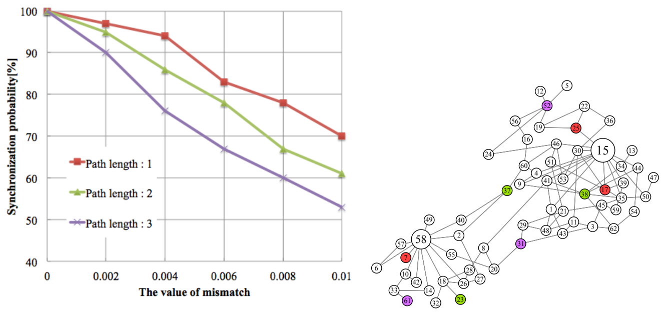
Fig. 5. Simulation result.

Figure 5 shows the 5 types nodes with the value of mismatch M. In this figure, the red nodes express the node of 1 degree, the green nodes nodes express the node of 2 degrees, the purple nodes express the node of 3 degrees, the watery nodes express the node 4 degrees, and the blue nodes express the node of 5 degrees. Figure 6 shows the change of synchronization probability when we add the mismatch as Fig. 5. The colors of each line is corresponding to the colors of each node in Fig. 5. In Fig. 6, the vertical axis denotes synchronization probability, and the horizontal axis denotes the value of mismatch (M). 5 colors lines express degree-by-degree passage of the synchronization probability.

When three nodes have 4 degrees, we can confirm the synchronization probability maintains the highest condition. On the other hand, when three nodes have 1, we can confirm the synchronization probability maintains the lowest condition.

From this result, we can confirm degree of the node including larger mismatch affects the synchronization probability. Furthermore, the synchronization probability is affected from not only the size of degrees but also the location of the nodes with larger mismatch M. Following this result, we consider it is important to investigate about the effect of the location of the nodes with larger mismatch M on the synchronization probability in the network.

2) Effect of the path length from hubs to a number of nodes with M: In this section, we investigate in what way the synchronization probability in complex network draw influence from the path length from hubs to a number of nodes with larger mismatch M. The value of path length (1 : 3) expresses the shortest path from the hubs to a number of nodes with M. In this simulation, we investigate the passage of synchronization probability by changing the value of path length (1 : 3) from hubs to a number of nodes with M, and adding M from 0.002 to 0.01 in a number of nodes.



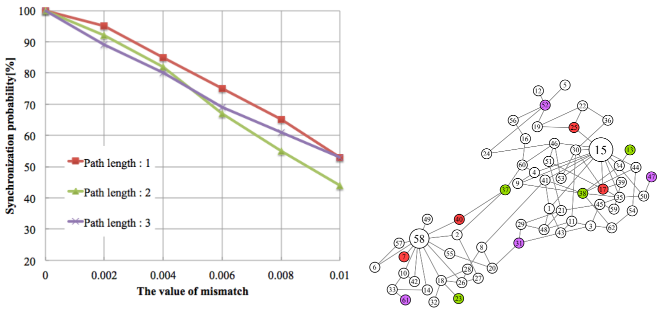
(a) Synchronization probability.

(b) The location of the nodes with M.

Fig. 6. Simulation result (M : 3 nodes).

In Fig. 6, we add M in three nodes. Three lines are corresponding to the value of the path length from the hubs to the three nodes with M, and the colors of each line is corresponding to the colors of each node. As a result, we can confirm the synchronization probability of each line increase with decreasing the value of the path length from the hubs to the three nodes with M. From this result, we can consider hubs are dispersing the effect of larger mismatch, and performing a function as maintaining the synchronous state. In order to make generalization this consideration, we investigate the synchronization probability by increasing the nodes with M.

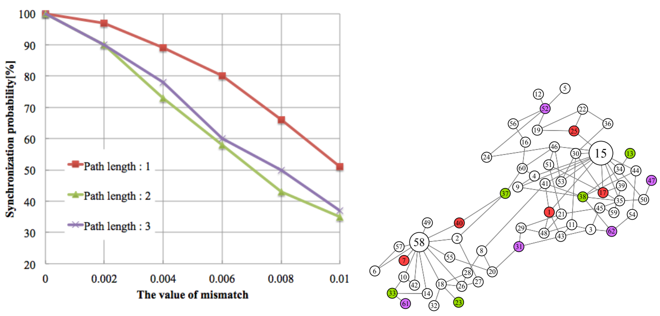
Next, synchronization probability changes as shown in Fig. 7 by adding M to four nodes. In this result, relative merits between path length : 2 and path length : 3 change from



(a) Synchronization Probability.

(b) The location of the nodes with M.

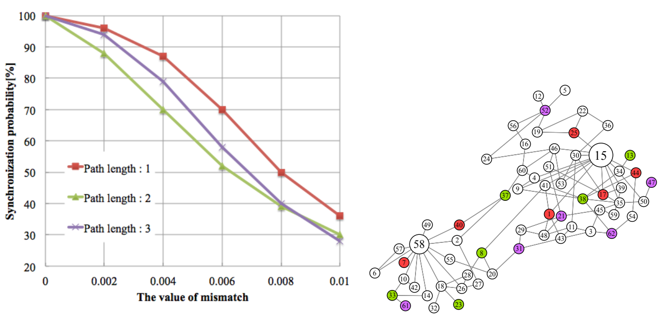
Fig. 7. Simulation result (M : 4 nodes).



(a) Synchronization Probability.

(b) The location of the nodes with M.

Fig. 8. Simulation result (M : 5 nodes).



(a) Synchronization Probability.

(b) The location of the nodes with M.

Fig. 9. Simulation result (M : 6 nodes).

M = 0.006. However, synchronization probability maintains the highest condition when path length is 1. Similarly, by increasing the number of nodes with M from 5 to 6, changes of the synchronization probability are obtained as shown in Fig. 8 and Fig. 9.

From these results, we can confirm the influence of the nodes with M can be dispersed by hubs when the value of the path length from hubs to a number of nodes with larger mismatch M is 1.

V. Conclusion

In this study, in order to investigate the influence of regional change in synchronization of complex network, we focus on the relationship between the structural metrics in a subset of the nodes is included larger mismatch and synchronization in complex network. In order to research this relationship, we investigate synchronization in complex network by changing structural metrics (degree and path length) of the nodes corresponding to the subset of the nodes adding larger mismatch than other nodes.

First, we investigate the synchronization probability in the network by changing the degree of the three nodes adding larger mismatch. Next, we investigate the synchronization probability in the network by changing the path length from hubs to a number of nodes with larger mismatch M.

We could observe the various change of the synchronization probability. Furthermore, we could consider the hub disperse the influence of larger mismatch, and perform a function as maintaining the synchronous state.

Acknowledgment

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