Switching Synchronization States of Ring System of Chaotic Circuit Including Time Delay in One-Direction

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***Abstract*—In this study, we investigate the novel coupling model and synchronization states observed in coupled time-delayed chaotic circuits. Moreover, we compare the standard coupling model and the novel coupling model and investigate the transition of chaos. The novel coupling model is a ring of coupled chaotic circuits with one-direction delay effects. The synchronization state of the novel coupling model is switching synchronization state. The switching synchronization state is changed by time delay of each subcircuit. We focus on relationships between switching synchronization state and the pattern of time delay. Finally, we investigate the cycle of switching synchronization state.**

I. Introduction

Studies on synchronization state are extensively carried out in various fields [1]-[3]. Recently, in particular, synchronization states in chaotic oscillators have been studied by many researchers. The behavior of chaotic oscillators is interesting. Then, chaos phenomena are quite dependent on initial values and not periodical and predictable. Moreover the synchronization states have caused very interesting phenomena. Synchronization and the related bifurcation of chaotic systems are good methods to describe various high-dimensional nonlinear phenomena in the field of natural science. However, many synchronization states of coupled chaotic oscillators have not been solved yet. The synchronization phenomena in electric circuit make clear the mechanism of the synchronization phenomena in our daily life. There are many nonlinear systems containing time delay, such as neural networks, control systems, meteorological systems, biological systems and so on in the natural world. Namely, it is considered that investigation of stability in such time-delay systems is significant [4]. Generation of chaos in time delayed system is reported self excited oscillation system containing time delay [5]. The oscillators have feedback systems which control gains in this study. This chaotic circuit can be easily realized by using simple electric circuit element and analyzed exactly. The coupling switch connects alternately with one subcircuit and the other with a fixed time interval. On the other hand, there are examples of nonlinear phenomena, chaotic synchronization and so on [6]. In particular, many studies on synchronization of coupled chaotic circuits have been reported [7].

In this study, we devise the novel coupling model that takes advantage of features of the time-delayed chaotic circuit. The novel coupled model is utilizing the characteristics of the circuit having time-delayed feedback. Then, we observe switching synchronization state. We carry out computer calculations for three coupled auto gain controlled oscillators containing time delay and investigate time delay of subcircuits effects a change of synchronization state and the time waveform.

II. Time-Delayed Chaotic Circuit

Figure 1 shows the time-delayed chaotic circuit. This circuit consists of one inductor L, one capacitor C, one linear negative resistor $-g$ and one linear positive resistor G of which amplitude is controlled by the switch containing time delay. The current flowing through the inductor L is i, and the voltage between the capacitor C is v. The circuit equations are normalized as Eqs. (1) and (2) by changing the variables as below.

(A) In case of switch connected to $-g$,

$$\left\{ \begin{array}{lcl} \dot{x} & = & y \\ \dot{y} & = & 2\alpha y - x, \end{array} \right. \tag{1}$$

(B) In case of switch connected to G,

$$\left\{ \begin{array}{lcl} \dot{x} & = & y \\ \dot{y} & = & -2\beta y - x. \end{array} \right. \tag{2}$$

By changing the parameters and variable as follow:

$$i = \sqrt{\frac{C}{L}} V_{th} x,\ v = V_{th} y,\ t = \sqrt{LC}\tau,$$
$$g\sqrt{\frac{C}{L}} = 2\alpha \text{ and } G\sqrt{\frac{C}{L}} = 2\beta.$$

The switching operation is shown in Fig. 2, it controls the amplitude of the oscillator. This switching operation is included time delay. T_d denotes the time delay. First, the switch is connected to a negative resistor. In state of that, the voltage v is amplified up to while v is oscillating, the amplitude exceeds the threshold voltage V_{th} which is the threshold control loop. Second, the system memorize the time as T_{th} while v is exceeding the threshold voltage V_{th} and that state is remained for T_{th}. In subsequent the instant of exceeding threshold V_{th}, the switch stays the state for T_d. After that switch is connected to positive resistor during T_{th}. The switch does not immediately connect in the positive resistor however the switch is connected after T_d. A set of switching operations control the amplitude of v. By using mapping method to this circuit, we could derive the one-dimensional Poincare map explicitly from each circuit, and the Poincare map was proved to have a positive Liapunov number with computer assistances [3].

III. Ring Coupled Time-Delayed Chaotic Circuit

In this section, we investigate the synchronization state for three time-delayed chaotic circuits coupled by different elements. Figure 3 shows a schematic of the three coupled time-delayed chaotic circuits. Two cases of interest are considered: the coupling elements are resistors R and inductors L_0. We change the parameters and variables as follows.

$$i_n = \sqrt{\frac{C}{L}} V_{th} x_n,\ v_n = V_{th} y_n,\ t = \sqrt{LC}\tau,\ g\sqrt{\frac{C}{L}} = 2\alpha,$$
$$G\sqrt{\frac{C}{L}} = 2\beta,\ \gamma_R = R\sqrt{\frac{C}{L}} \text{ and } \gamma_{L_0} = \frac{L}{L_0}.$$

The normalized circuit equations of the system are given as follows:

(A) In the case that the switch is connected to the negative resistor

$$\begin{cases} \dot{x}_n &= y_n \\ \dot{y}_n &= -x_n + 2\alpha y_n + \gamma_m (x_{n-1} - 2x_n + x_{n+1}) \end{cases} \tag{3}$$

(B) In the case that the switch is connected to the positive resistor

$$\begin{cases} \dot{x}_n &= y_n \\ \dot{y}_n &= -x_n - 2\beta y_n + \gamma_m (x_{n-1} - 2x_n + x_{n+1}) \end{cases} \tag{4}$$

where $n = 1, 2, 3$, $m = R, L_0$ and $x_0 = x_3$, $x_4 = x_1$.

A. Coupled by resistors R

Figure 5 shows some simulation results, in which the quasi-in-phase synchronization state can be observed. When the coupling strength γ_R is larger than 0.1, full in-phase synchronization can be observed. However, full in-phase synchronization cannot be observed or synchronization is lost in the case of a small γ_R. The Poincare section in Fig.5(d) shows the presence of chaos.

B. Coupled by inductors L_0

Figure 6 shows some simulation results in the case of time-delayed chaotic circuits coupled by inductors L_0. In-phase synchronization state and three-phase synchronization state can be observed when γ_{L_0} is smaller than 0.1. When γ_{L_0} sets 0.01, three-phase and in-phase are switched. When γ_{L_0} sets 0.5, only three-phase synchronization state can be observed. The Poincare section in Fig.6(d) shows the presence of chaos.

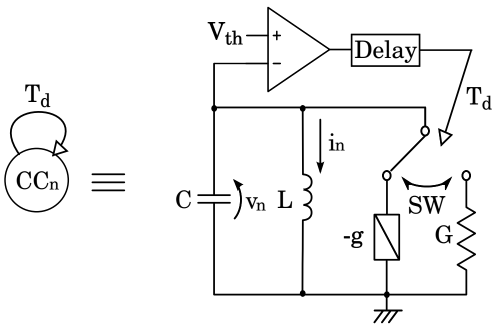


Fig. 1. Time-delayed chaotic circuit.

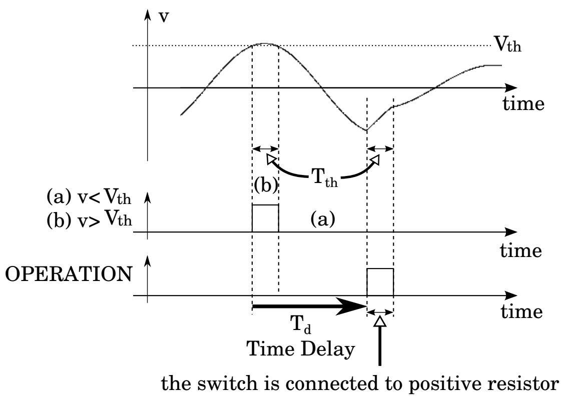


Fig. 2. Switching operation.

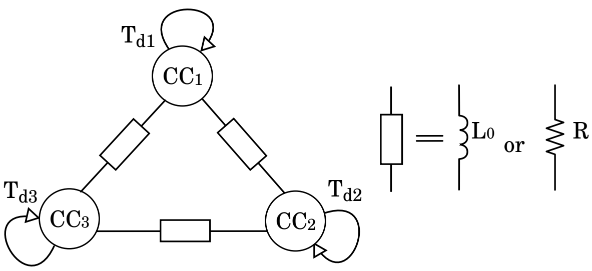


Fig. 3. Ring coupling model.

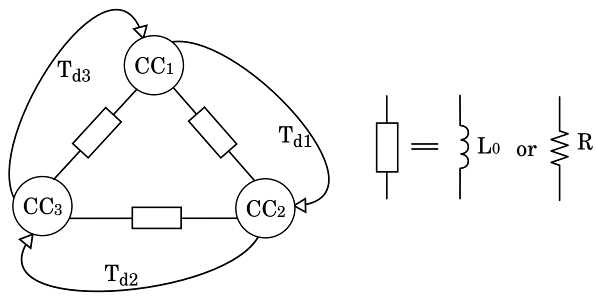
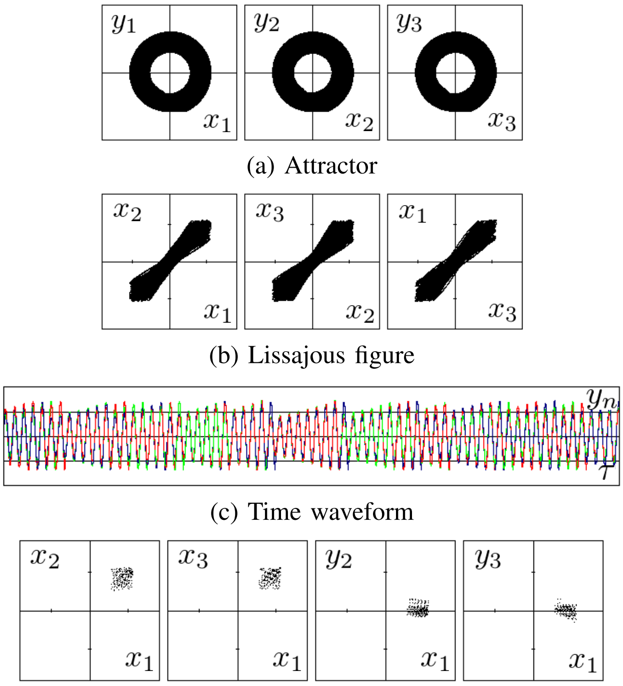
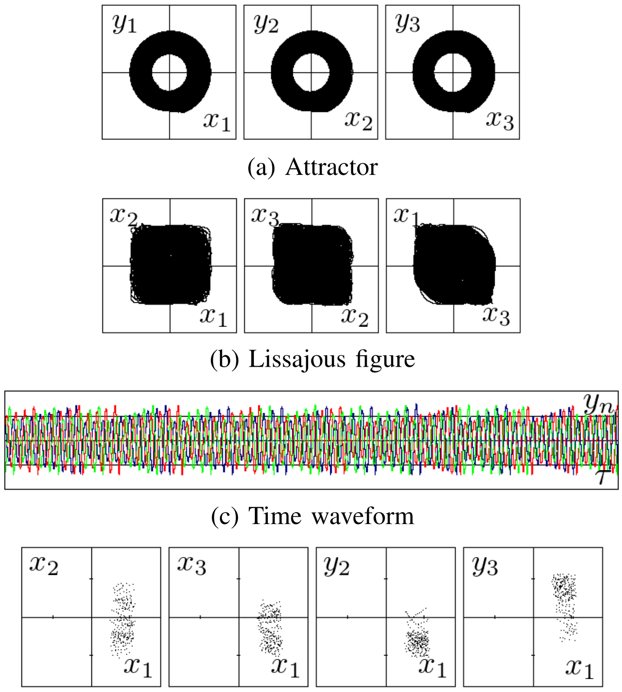


Fig. 4. System including time delay in one direction.



(a) Attractor
(b) Lissajous figure
(c) Time waveform
(d) Poincare section

Fig. 5. Simulation results of system coupled by resistors R. of $\alpha = 0.015$, $\beta = 0.5$, $\gamma_R = 0.01$ and $T_{dn} = \pi$.



(a) Attractor
(b) Lissajous figure
(c) Time waveform
(d) Poincare section

Fig. 6. Simulation results of system coupled by inductors L_0. $\alpha = 0.015$, $\beta = 0.5$, $\gamma_{L_0} = 0.01$ and $T_{dn} = \pi$.

IV. System Including Time Delay in One Direction

The circuit in this study has characteristic time delays methods. We have devised coupled systems as shown in Fig. 4. It is called coupled systems and "a ring of coupled chaotic circuits with one-direction delay effects" The normalized circuit equations of this system are the same as Eqs.(3) and (4). It is different pnly how to use a time-delayed signal. The novel coupling model has three time delays T_{dn}.

A. Coupled by resitors R

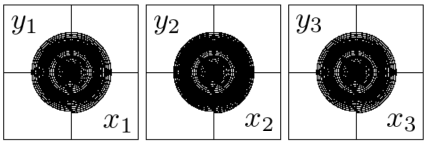
We use resistors R as coupling elements. The simulation results are shown in the Fig. 7. The time waveform in Fig.7(a) shows in-phase synchronization and the amplitude of y_n is switched sequentially. This phenomenon is defined as the switching synchronization state. There are three definitions of switching synchronization state. First, maintain a constant phase difference. Second, amplitude oscillations in the order. Third, patterns of amplitude oscillation. However, when γ_R is larger than 0.1, the switching synchronization state is lost and a full in-phase synchronization state can be observed. Figure 7(d) shows the presence of small chaos. The chaos of Fig. 7(d) is changing compared to Fig. 5(d) by one-direction delay effects. Changing the time delays T_{dn} change the cycle of the time waveform. Figure 9 shows the cycle of switching synchronization with symmetric or asymmetric delay. The variation of cycle in asymmetric delay is less than symmetric delay.

B. Coupled by inductors L_0

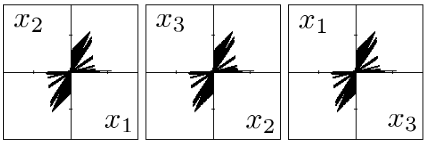
When we use inductors L_0 as coupling elements, the simulation results are shown in the Fig. 8 can be observed. The time waveform in Fig.8(a) shows a phase difference and the amplitude of y_n is switched sequentially. The switching synchronization state with phase difference can be observed. The amplitude alternately diverges and converges with different divergence and convergence times. However, when γ_{L_0} is larger than 0.1, the switching synchronization state is lost. The presence of small chaos is shown Fig. 8(d). The chaos of Fig. 8(d) is changing compared to Fig. 6(d) by one-direction delay effects. Changing the time delays T_{dn} change the the cycle of the time waveform. Figure 9 shows cycle of switching synchronization with symmetric or asymmetric delay. The variation of cycle in asymmetric delay is less than symmetric delay. Moreover, when compared with the coupling element, the number of cycles is larger for system coupled by resistors R.

V. Conclusion

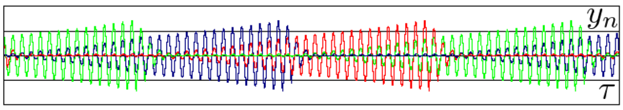
In this study, we devised coupled systems that take advantage of features of the time-delayed chaotic circuit. We investigated the synchronization state of a ring of coupled chaotic circuits with one-direction delay effects. As a result, some special synchronization states can be observed. This is the switching synchronization state. The switching synchronization state. There are existence or non-existence of phase difference in the switching synchronization state. Moreover,



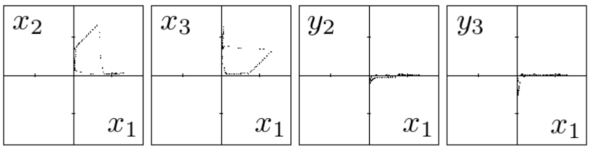
(a) Attractor



(b) Lissajous figure

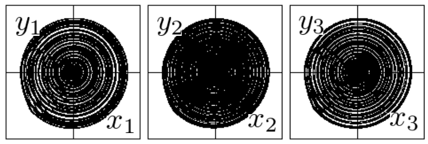


(c) Time waveform

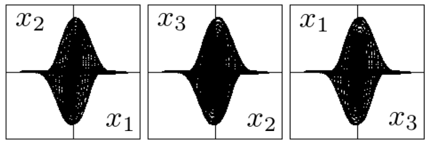


(d) Poincare section

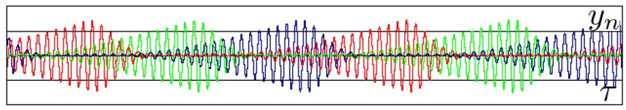
Fig. 7. Simulation results of system coupled by resistors R. $\alpha = 0.015$, $\beta = 0.5$, $\gamma_R = 0.01$ and $T_{dn} = \pi$.



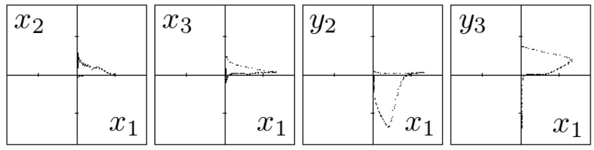
(a) Attractor



(b) Lissajous figure



(c) Time waveform



(d) Poincare section

Fig. 8. Simulation results of system coupled by inductors L_0. $\alpha = 0.015$, $\beta = 0.5$, $\gamma_{L_0} = 0.01$, $T_{dn} = \pi$.

when time delay of subcircuits changes, the cycle of switching synchronization state changes. The switching of the amplitude can be observed by difference of time delay. The variation of cycle in asymmetric delay is less than symmetric delay.

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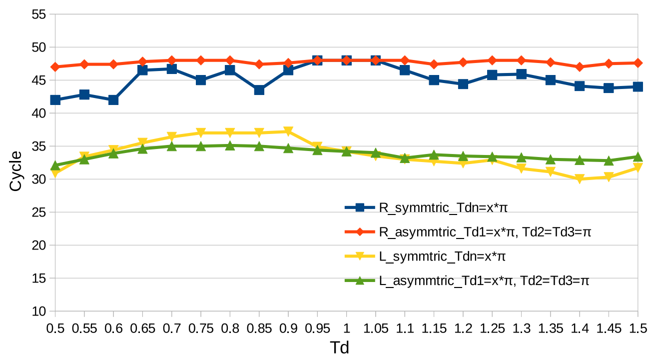

Fig. 9. Cycle of switching synchronization coupled by R or L_0 with symmetric or asymmetric delay.

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