

# Firefly Algorithm combined with chaotic map

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**Abstract**—Recently, nature-inspired metaheuristic optimization algorithms such as Firefly Algorithm (FA) has been developed. FA is idealized from the social behavior of fireflies based on their flashing characteristics. Furthermore, FA combined with chaotic map is shown to be of benefit. In this study, we propose an algorithm combined FA with Bernoulli shift map and Tent map. Compared to the previous study, we investigate a difference approach to insert chaotic map. We compare improved FA to the conventional FA using 7 benchmark functions of Congress on Evolutionary Computation 2013. In our study, improved FA performs better than the conventional FA.

## I. INTRODUCTION

Swarm Intelligence is one of research territory of artificial intelligence. Examples existing in nature are ant, bee, bird and fish. The swarm intelligence technique is important because simple control regulation is more beneficial than complex control regulation. The applications of swarm intelligence technique are unmanned aircraft and a self-driving car. The good points of this technique are not being high altitude and being able to be downsize of robots.

The non-linearity of many optimization problems often results in local optima. To overcome this issue, metaheuristic optimization algorithms are used. These optimization algorithms attempt to idealize social behavior or natural phenomena. Several metaheuristic optimization algorithms are developed for global search. Such optimization algorithms develop more efficiency and solve larger problems. Metaheuristic algorithms have Genetic Algorithm (GA), Ant Colony Optimization (ACO), Particle Swarm Optimization (PSO), Firefly Algorithm (FA), etc. In our study, we use the FA.

FA idealizes the social behavior of fireflies based on their flashing characteristics. In social insect colonies, each individual insect seems to have its own agenda and the group in total appears to be highly organized. Nature-inspired Algorithms have been demonstrated to show effectiveness and efficiency to solve difficult optimization problems. A swarm is a group of multi-agent systems such as fireflies. Simple agents coordinate their activities to solve the complex problem to multiple forage sites in dynamic environments. Therefore, FA is applicable for mixed variable and engineering optimization.

Recent applications of nonlinear dynamics, especially of chaos, have drawn attention in many fields. Chaos is seemingly a random movement of deterministic system. Chaos system has the properties of sensitivity to initial conditions. Therefore, using chaotic system in image encryption can meet security requirements. Moreover, the chaotic encryption algorithms

utilize one-dimensional chaos map, multi-dimensional chaos map and ultra-dimensional chaos map. We drew attention to one-dimensional chaotic map. Furthermore, FA combined with chaotic map is shown to be of benefit. In this study, we propose an algorithm combined FA with Bernoulli shift map and Tent map. Compared to the previous study, we investigate a difference approach to insert chaotic map into the conventional FA.

This paper illustrates FA combined with chaotic map. Section 2 describes the conventional FA. Section 3 explains our proposed method. Numerical simulation and simulation results are discussed in Section 4. Section 5 is discussed the conclusion and outlines directions for further research.

## II. FIREFLY ALGORITHM

First, we will introduce the behavior of fireflies. Firefly is the one of insects. The flashing light of fireflies is an amazing sight in the summer sky in the tropical and temperate regions. There are about 2000 firefly species in the world, and Japan has about 40 firefly species. Moreover, most fireflies produce short and rhythmic flashes. The pattern of flashes is often unique for a particular species. Some species of fireflies can even synchronize their flashes.

The light intensity at a particular distance  $r$  from the light source obeys the inverse square law. Furthermore, the air absorbs light which becomes weaker and weaker as the distance increases.

We can idealize some of the flashing characteristics of fireflies so as to develop firefly-inspired algorithms. The conventional FA was developed by Xin-She Yang in 2007. We use the following 3 idealized rules:

- All fireflies are unisex so that one firefly will be attracted to other fireflies regardless of their sex.
- Attractiveness is proportional to the their brightness, thus for any two flashing fireflies, the less brighter one will move towards the brighter one. The attractiveness is proportional to the brightness and they both decrease as their distance increases. If there is no brighter one than a particular firefly, it will move randomly.
- The brightness of a firefly is affected or determined by the landscape of the objective function.

The attractiveness of firefly  $\beta$  is defined by

$$\beta = \beta_0 e^{-\gamma r_{ij}^2} \quad (1)$$

where  $\gamma$  is the light absorption coefficient,  $\beta_0$  is the attractiveness at  $r_{ij} = 0$  and  $r_{ij}$  is the distance between any two fireflies  $i$  and  $j$  located at  $x_i$  and  $x_j$  respectively. The firefly  $i$  is attracted to another more attractive firefly  $j$  and the movement of firefly  $i$  is determined by

$$\mathbf{x}_i = \mathbf{x}_i + \beta(\mathbf{x}_j - \mathbf{x}_i) + \alpha\epsilon_i \quad (2)$$

where  $\alpha$  is the randomization parameter and  $\epsilon_i$  is a random vector which are drawn from a Gaussian distribution.

The parameter  $\alpha(t)$  is defined by

$$\alpha(t) = \alpha(0) \left( \frac{10^{-4}}{0.9} \right)^{t/t_{max}} \quad (3)$$

where  $t$  is the number of iteration.  $t_{max}$  is the maximum number of  $t$ .

### III. OUR PROPOSED METHOD

We propose the improved FA(BFA and TFA). BFA and TFA are combined the conventional FA with Bernoulli shift map and Tent map. These maps are one of the one-dimensional chaotic maps, which is the simplest systems with the capability of generating chaotic motion. One-dimensional maps are introduced as follows. It generates chaotic sequences in  $(0, 1)$  assuming Eq. (5) and Eq. (6). In the previous study, the author inserted 12 chaotic maps into the attractiveness of firefly  $\beta$ . Moreover the author inserted 12 chaotic maps into the light absorption coefficient  $\gamma$ . Each method is simulated by Sphere Function and Griewank's Function. In this study, we insert Bernoulli shift map and Tent map into the vector of random variable.

$$\mathbf{x}_i = \mathbf{x}_i + \beta(\mathbf{x}_j - \mathbf{x}_i) + \alpha(\epsilon_i + \mathbf{z}_i) \quad (4)$$

The Bernoulli shift map(Fig. 1) belongs to the class of piecewise linear maps similar to the logistic map or the Kent map. It is formulated as follows:

$$z_{i+1} = \begin{cases} 2z_i & (0 \leq z_i \leq 0.5) \\ 2z_i - 1 & (0.5 \leq z_i \leq 1). \end{cases} \quad (5)$$

The Tent map(Fig. 2) is similar to the logistic map. It displays some specific chaotic effects. This map is formulated as follows:

$$z_{i+1} = \begin{cases} 2z_i & (0 \leq z_i \leq 0.5) \\ 2 - 2z_i & (0.5 \leq z_i \leq 1). \end{cases} \quad (6)$$

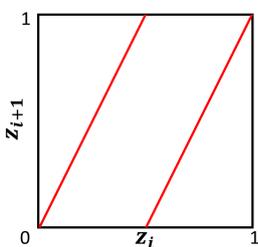


Fig. 1. Bernoulli shift map.

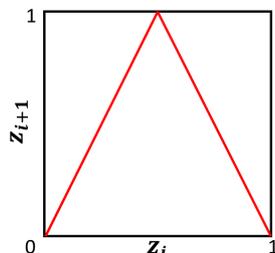


Fig. 2. Tent map.

### IV. NUMERICAL SIMULATION

We compare BFA and TFA to the conventional FA using 7 benchmark functions of Congress on Evolutionary Computation(CEC) 2013. Table I shows the functions we used. We chose 2 unimodal functions( $f_1$  and  $f_2$ ), 3 basic multimodal functions( $f_3$ ,  $f_4$  and  $f_5$ ) and 2 composition functions( $f_6$  and  $f_7$ ).

In this simulation, the optimal solutions  $x^*$  of these benchmark functions are shifted from 0 and the global optima  $f(x^*)$  are not equal to 0. In addition, we assign the search range of these function is  $[-100, 100]^D$  ( $D$ :Dimension), the number of firefly  $N$  is 30. Each numerical experiment is run 50 times. Furthermore, we use  $\beta_0 = 1.0$ ,  $\gamma = 1.0$  and  $t_{max} = 1500$ . Table II shows the average, minimum and maximum error value.

TABLE I  
2013 CEC BENCHMARK FUNCTIONS

No.	Name	$f(x^*)$
1	Sphere Function	-1400
2	Rotated Discus Function	-1100
3	Rotated Rosenbrock's Function	-900
4	Rotated Weierstrass Function	-600
5	Rotated Griewank's Function	-500
6	Composition Function 2 (n=3, Unrotated)	800
7	Composition Function 5 (n=3, Rotated)	1100

We chose these functions because these functions is popular function for optimization problems.

Sphere Function is no dependency between variables. Weierstrass Function is real-valued function advanced in 1872 by Karl Weierstrass. This function is continuous. However it is almost Nondifferentiable. Historically, this funtion is very important as exaple of pathological functions. Griewank's Function is almost gentle globally. However it has many local minima. Griewank's Function is said to be suitable for Simulated Annealing(SA) search. Composition Function 2 is generated from Schwefel's Function. Moreover Composition Function 5 is generated from Rotated Schwefel's Function, Rotated Rastrigin's Function and Rotated Weierstrass Function. Moreover we tried to use these functions for FA.

Showing in Table II, BFA and TFA performs better than the conventional FA. Both BFA and TFA performs better than FA in  $f_1$ ,  $f_2$ ,  $f_4$  and  $f_5$ . In the unimodal functions( $f_1$  and  $f_2$ ), TFA is the best performance. In the basic multimodal functions( $f_3$ ,  $f_4$  and  $f_5$ ), BFA is the best performance. In the composition functions( $f_6$ ), TFA is better than FA and BFA. In the composition functions( $f_7$ ), the result changed relatively little. Therefore, We assumed BFA and TFA is effective.

In this simulation, we compare the average error values of simulation results. In results, BFA performs better than FA on 5 functions( $f_1$ ,  $f_2$ ,  $f_3$ ,  $f_4$  and  $f_5$ ). TFA performs better than FA on 5 functions( $f_1$ ,  $f_2$ ,  $f_4$ ,  $f_5$  and  $f_6$ ).

TABLE II  
SIMULATION RESULTS

$f$		FA	BFA	TFA
$f_1$	avg	$6.45 \times 10^{-4}$	<b><math>6.26 \times 10^{-4}</math></b>	<b><math>6.11 \times 10^{-4}</math></b>
	min	$4.32 \times 10^{-4}$	$3.27 \times 10^{-4}$	$3.91 \times 10^{-4}$
	max	$9.66 \times 10^{-4}$	$1.07 \times 10^{-3}$	$8.97 \times 10^{-4}$
$f_2$	avg	$1.22 \times 10^5$	<b><math>1.19 \times 10^5</math></b>	<b><math>1.14 \times 10^5</math></b>
	min	$7.55 \times 10^4$	$6.61 \times 10^4$	$7.12 \times 10^4$
	max	$2.14 \times 10^5$	$1.86 \times 10^5$	$1.68 \times 10^5$
$f_3$	avg	$2.73 \times 10^1$	<b><math>2.72 \times 10^1</math></b>	$2.73 \times 10^1$
	min	$2.54 \times 10^1$	$2.53 \times 10^1$	$2.59 \times 10^1$
	max	$2.85 \times 10^1$	$2.83 \times 10^1$	$2.90 \times 10^1$
$f_4$	avg	$1.04 \times 10^1$	<b><math>9.86 \times 10^0</math></b>	<b><math>1.03 \times 10^1</math></b>
	min	$7.07 \times 10^0$	$3.68 \times 10^0$	$5.69 \times 10^0$
	max	$1.59 \times 10^1$	$1.73 \times 10^1$	$1.50 \times 10^1$
$f_5$	avg	$5.63 \times 10^{-1}$	<b><math>4.06 \times 10^{-1}</math></b>	<b><math>5.39 \times 10^{-1}</math></b>
	min	$4.15 \times 10^{-2}$	$7.28 \times 10^{-2}$	$3.08 \times 10^{-2}$
	max	$2.23 \times 10^0$	$1.70 \times 10^0$	$1.93 \times 10^0$
$f_6$	avg	$3.31 \times 10^3$	$3.46 \times 10^3$	<b><math>3.11 \times 10^3</math></b>
	min	$6.08 \times 10^2$	$1.36 \times 10^3$	$1.48 \times 10^3$
	max	$6.27 \times 10^3$	$6.13 \times 10^3$	$6.21 \times 10^3$
$f_7$	avg	<b><math>2.33 \times 10^2</math></b>	<b><math>2.33 \times 10^2</math></b>	$2.34 \times 10^2$
	min	$2.01 \times 10^2$	$2.19 \times 10^2$	$2.14 \times 10^2$
	max	$2.53 \times 10^2$	$2.50 \times 10^2$	$2.53 \times 10^2$

## V. CONCLUSION

This paper introduced the improved Firefly Algorithm(BFA and TFA). We tried to improve the conventional FA using Bernoulli shift map and Tent map. We compared average error values of simulation results. BFA and TFA performed better than the conventional FA. Furthermore, we assumed BFA and TFA is effective.

In the future work, we will investigate BFA and TFA using more functions and insert other chaotic maps. Furthermore, we will compare BFA and TFA to FA inserted only chaotic maps.

## REFERENCES

- [1] A.H. Gandomi, X.-S. Yang, S. Talatahari, A.H.Alavi, "Firefly algorithm with chaos," Commun Nonlinear Sci Numer Simulat 18 (2013) 89-98.
- [2] X.-S. Yang, "Nature-Inspired Metaheuristic Algorithms Second Edition," Luniver Press(2010).
- [3] J.J.Liang, B.Y.Qu, P.N.Suganthan, Alfredo G. Hernandez-Daz, "Problem Definitions and Evaluation Criteria for the CEC2013 Special Session on Real-Parameter Optimization," Technical Report 201212, Computational Intelligence Laboratory, Zhengzhou University, Zhengzhou ChinaAndTechnical Report, Nanyang Technological University, Singapore(2013).
- [4] J.Senthilnath, S.N.Omkar, V.Mani, "Clustering using firefly algorithm: Performance study," Swarm and Evolutionary Computation 1 (2011) 164-171.
- [5] Amir Hossein Gandomi, Xin-She Yang, Amir Hossein Alavi, "Mixed variable structural optimization using Firefly algorithm," Computers and Structures 89 (2011) 2325-2336.