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Synchronization Phenomena in Two Rings of Coupled Three van der Pol Oscillators

Daiki Nariai, Tran Minh Hai, Yoko Uwate and Yoshifumi Nishio
Dept. Electrical and Electronic Eng.,
Tokushima University,
2-1 Minami-Josanjima,
E-mail: {nariai, minhhai, uwate, nishio}@ee.tokushima-u.ac.jp

***Abstract*—In this study, we investigate synchronization phenomena in two rings of van der Pol oscillators coupled by resistors. We propose novel coupled oscillatory system such as two rings of van der Pol oscillators coupled by resistors. We focus on coupling strength of coupled van der Pol oscillators. From computer simulations, we investigate how synchronization phenomena changes by changing the coupling strength. We observe various synchronization phenomena by changing the coupling strengths. We demand to control the synchronization phenomena by changing the coupling strength.**

I. Introduction

Synchronization phenomena of coupled oscillators is the most familiar phenomena. Among them, synchronization phenomena of van der Pol oscillators is similar to natural phenomena. Because it is easy to observe the synchronization phenomena of van der Pol oscillators by changing frequency. Synchronization phenomena of van der Pol oscillator has been studied in various fields since ancient times, such as in electrical systems, in mechanical systems, in biological systems and basically everywhere. For example, firefly luminescence, heartbeat, and so on. We explain synchronization phenomena with concrete descriptions in heatbeat. Heat pulsative in a normal way because all cardiac muscle cell synchronize. Many researchers have proposed various coupled oscillatory networks of van der Pol oscillators. We focus on coupling strength of coupled oscillatory networks consisted of two kinds of van der Pol oscillatiors.

The van der Pol oscillator is simple ciruit. It is consisted of resistor, inductor, capacitor and nonlinear resistor. It was invented by electrical engineer Balthasar van der Pol. Equation of van der Pol is second-order differential equation.

In this study, we propose novel coupled oscillatory system such as two rings of van der Pol oscillators coupled by resistors. First ring is consisted of three van der Pol oscillators connected by resistors. Second ring is consisted of three van der Pol oscillators connected by inductors. By computer simulations, we investigate synchronization phenomena observed in the proposed circuit system by changing the coupling strength.

II. System model

Figure 1 shows two circuits which were used in my research. We use six van der Pol oscillators (three VDP1 and three VDP2). Figure 2 shows a system model with van der Pol oscillators (VDP1 and VDP2). We use two ring circuits of van der Pol oscillators, three VDP1 of first ring are connected by resistors, three VDP2 of second ring are connected by inductors. First and second ring are connected by resistor (R_1, R_2, R_3). We observe synchronization phenomena of adjacent oscillators. We investigate synchronization phenomena how changing by changing the value of resistors.

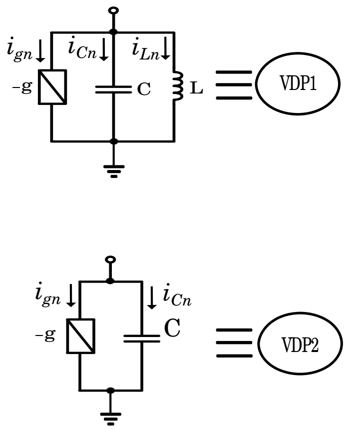

Fig. 1. Circuit of van der Pol oscillators.

The circuit equation of first ring are given as follows:

$$\begin{cases} -i_{gn} - i_{cn} - i_{Ln} & = & i_{an} + i_{bn} + i_{rn} \\ L\frac{di_{Ln}}{dt} & = & v_n \\ v_n - v_i & = & (i_{an} - i_{b(i)})R \\ v_n - v_j & = & (i_{bn} - i_{a(j)})R \\ v_n - v_{n+3} & = & (i_{rn} - i_{r(n+3)})R \end{cases} \quad (1)$$

The circuit equation of second ring are given as follows:

$$\begin{cases} -i_{gn} - i_{cn} & = & i_{an} + i_{bn} + i_{rn} \\ v_n - 2L\frac{di_{an}}{dt} & = & -(i_{an} + i_{b(i)})R' \\ v_n - 2L\frac{di_{bn}}{dt} & = & -(i_{bn} + i_{a(j)})R' \\ v_n - v_{n-3} & = & (i_{rn} - i_{r(n-3)})R \end{cases} \quad (2)$$

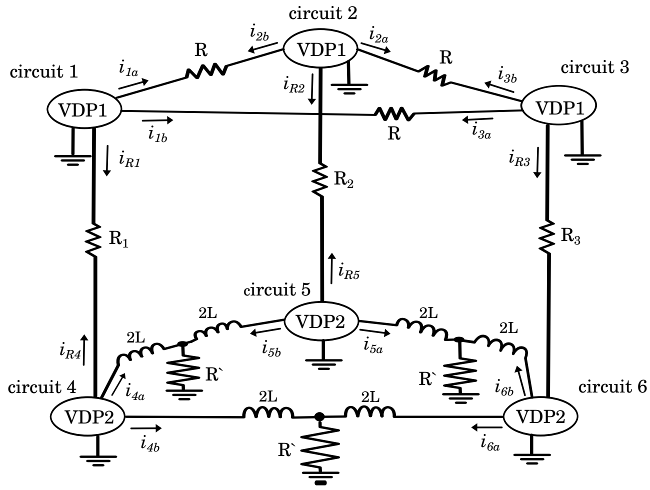

Fig. 2. Circuit model.

where n denotes the number of circuit and $n = 1, 2, 3, 4, 5, 6$. i denotes the number of circuit and $i = 2, 3, 1, 5, 6, 4$. j denotes the number of circuit and $j = 3, 1, 2, 6, 4, 5$.

Nonlinear resistor defined as follows:

$$i_{gn} \quad = \quad -g_1 v_n + g_3 v_n^3 \tag{3}$$

By changing the variables and parameters.

$$\begin{aligned} & t = \sqrt{LG}\tau, v_n = \sqrt{\tfrac{g_1}{3g_3}}x_n, i_n = \sqrt{\tfrac{g_1 C}{3g_3 L}}y_n, \\ & \varepsilon = g_1\sqrt{\tfrac{C}{L}}, \alpha = \tfrac{1}{R}\sqrt{\tfrac{L}{C}}, \\ & \beta = R'\sqrt{\tfrac{C}{L}}, \gamma_n = \tfrac{1}{R_n}\sqrt{\tfrac{L}{C}} \end{aligned} \tag{4}$$

The normalized equations of first ring are given as follows:

$$\begin{cases} \dot{x}_n &= \varepsilon(x_n - x_n^3) - y_n - \gamma(x_n - x_{n+3}) \\ & \quad +\alpha(-x_n + x_i + x_j) \\ \dot{y}_n &= x_n \end{cases} \tag{5}$$

The normalized equations of second ring are given as follows:

$$\begin{cases} \dot{x}_n &= \varepsilon(x_n - x_n^3) - y_{an} \\ & \quad -y_{bn} + \gamma(x_n - x_{n-3}) \\ \dot{y}_{an} &= x_n - \beta(y_{an} + y_{b(i)}) \\ \dot{y}_{bn} &= x_n - \beta(y_{bn} + y_{a(j)}) \end{cases} \tag{6}$$

where n denotes the number of VDP1 and VDP2, $n = 1, 2, 3, 4, 5, 6$. The parameters ε, α, β, and γ denote the coupling strengh of the inductor, resistor R, resistor R' and resistor R_n.

III. Simulation results

The simulation result of the system model are shown from Fig. 3 to Fig. 6. The value of the parameters are set to $\varepsilon = 0.05$, $\alpha = 0.05$, $\beta = 0.05$.

In case of $\gamma_1 = \gamma_2 = \gamma_3 = 0.02$, we conduct simulation, each one at a different initial value. In Fig. 3, synchronization phenomena is observed in circuit 1 and circuit 4. However, in Fig. 4, synchronization phenomena is observed in circuit 2 and circuit 5.

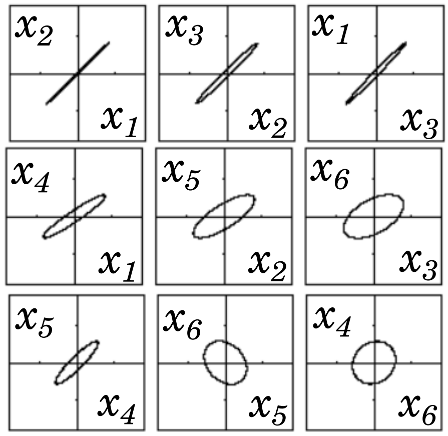

Fig. 3. Phase difference ($\gamma_1 = \gamma_2 = \gamma_3 = 0.02$)

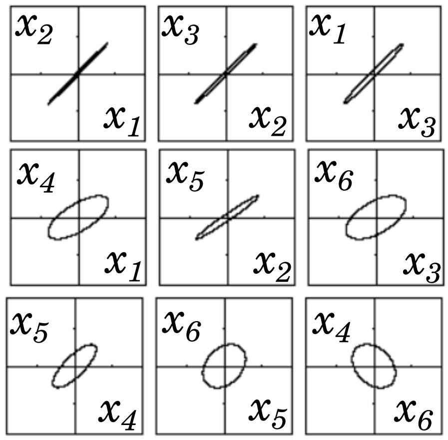

Fig. 4. Phase difference ($\gamma_1 = \gamma_2 = \gamma_3 = 0.02$)

By chaging γ_1, γ_2 and γ_3, we can control synchronization phenomena regardless of initial value.

In case of $\gamma_1 = 0.02, \gamma_2 = 0.005, \gamma_3 = 0.02$, synchronization phenomena is observed in circuit 4 and circuit 6 without reference to initial value. In this result, when we increase two of γ_1 and γ_3, three oscillators of first ring and two of three oscillators of second ring are synchronized.

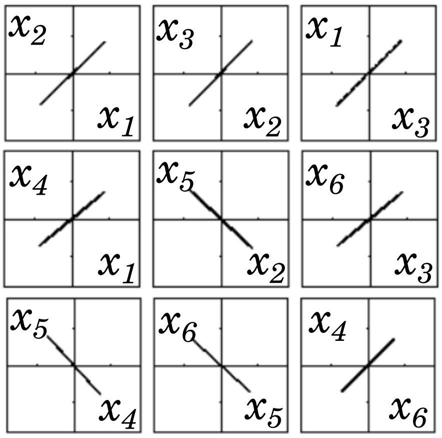

Fig. 5. Phase difference ($\gamma_1 = 0.02, \gamma_2 = 0.005, \gamma_3 = 0.02$)

In case of $\gamma_1 = 0.001, \gamma_2 = 0.0001, \gamma_3 = 0.02$, in-phase synchronization phenomena is observed in oscillators of first ring, 3-phase synchronization phenomena is observed in oscillators of second ring. In this result, when we increase one of γ_3, three oscillators of first ring become in phase synchronization phenomena, oscillators of second ring become 3-phase synchronization phenomena.

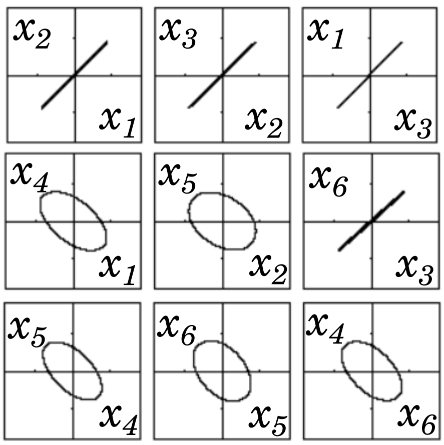

Fig. 6. Phase difference ($\gamma_1 = 0.001, \gamma_2 = 0.001, \gamma_3 = 0.02$)

We observe various synchronization phenomena by changing the coupling strengths. In this results, when we demand two oscillators of second ring to synchronize, we strengthened up two of γ_1, γ_2 and γ_3. When we demand three oscillators of second ring to become 3-phase synchronization phenomena, we strengthened up two of γ_1, γ_2 and γ_3. Therefore we can control synchronization phenomena by coupling strengths.

IV. Conclusions

We have proposed a system model using two rings of coupled three van der Pol oscillators coupled by resistors or inductors. We can control the synchronization phenomenon by varying the coupling strengths. When three coupling strengths (γ_1, γ_2, γ_3) equal, synchronization phenomena are observed by changing initial value. However when we increase two of γ_1, γ_2 and γ_3, three oscillators of first ring and two oscillators of second ring are synchronized. When we strengthened up one of γ_1, γ_2 and γ_3, oscillators of first ring become in-phase synchronization phenomena, oscillators of second ring become 3-phase synchronization phenomena. In the future, we investigate synchronization phenomena using other parameters and analyze my circuit model.

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