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Synchronization Phenomena in Two Rings of van der Pol Oscillators

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1. Introduction

Synchronization phenomena is the most familiar phenomena that exist in nature and it has been studied in various fields since ancient times, such as in electrical systems, in mechanical systems, in biological systems, basically every where.

In this study, we propose novel coupled oscillatory system such as two rings of van der Pol oscillators coupled by resistors. We investigate synchronization phenomena observed in the proposed circuit system by changing the coupling strength.

2. System model

Figure 1 shows a system model constituted van der Pol oscillators (VDP1 and VDP2). We use two rings of van der Pol oscillators, three VDP1 of first ring are connected by resistor, three VDP2 of second ring are connected by inductor. First and second ring are connected by resistor (R_1, R_2, R_3). We investigate synchronization phenomena how changing by changing the value of resistors.

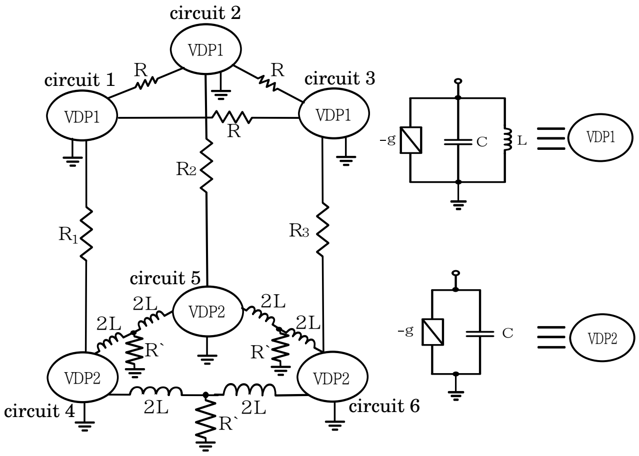

Figure 1: Circuit model.

The normalized equations of VDP1 circuit are given as follows:

$$\begin{cases} \dot{x_n} = \varepsilon(x_{1n} - x_{1n}^3) - y_{1n} \\ \qquad +\alpha(-x_{1n} + x_{1n+1} + x_{1n-1}) - \gamma(x_1 n - x_2 n) \\ \dot{y_n} = x_n. \end{cases} \quad (1)$$

The normalized equations of VDP2 circuit are given as follows:

$$\begin{cases} x_2\dot{n} = \varepsilon(x_n - x_n^3) - y_{an} - y_{bn} + \gamma(x_{2n} - x_{1n}) \\ y_a\dot{2}n = x_{2n} - \beta(y_{a2n} + y_{b2n+1}) \\ y_b\dot{2}n = x_{2n} - \beta(y_{b2n} + y_{a2n-1}). \end{cases} \quad (2)$$

where n denotes the number of VDP1 and VDP2, $n = 1, 2, 3$. The parameters ε, α, β, and γ denote the coupling strengh of the inductor, resistor R, resistor R' and resistor R_n.

3. Simulation results

The simulation result of the system model are shown from Fig 2 to Fig 5. The value of the parameters are set to $\varepsilon = 0.05$, $\alpha = 0.05$, $\beta = 0.05$. In case of $\gamma_1 = \gamma_2 = \gamma_3 = 0.02$, In second ring, synchronization phenomena change by initial value. By chaging γ_1, γ_2 and γ_3, we can control synchronization phenomena regardless of initial value. In case of $\gamma_1 = 0.02, \gamma_2 = 0.005, \gamma_3 = 0.02$, circuit 4 and circuit 6 become synchronization phenomena without reference to initial value. In case of $\gamma_1 = 0.001, \gamma_2 = 0.0001, \gamma_3 = 0.02$, oscillators of first ring become synchronization phenomena, oscillators of second ring become 3-phase synchronization phenomena. We observe synchronization phenomena by changing the coupling strengths.

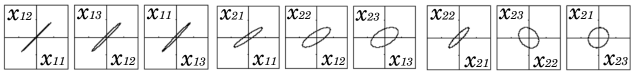

Figure 2: $\gamma_1 = \gamma_2 = \gamma_3 = 0.02$.

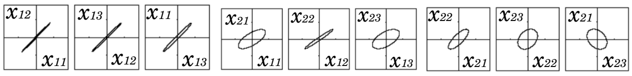

Figure 3: $\gamma_1 = \gamma_2 = \gamma_3 = 0.02$.

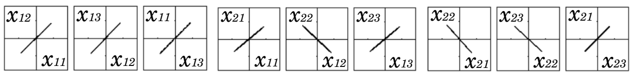

Figure 4: $\gamma_1 = 0.02, \gamma_2 = 0.005, \gamma_3 = 0.02$.

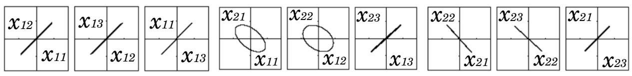

Figure 5: $\gamma_1 = 0.001, \gamma_2 = 0.001, \gamma_3 = 0.02$.

4. Conclusions

We have proposed a system model using two rings of coupled three van der Pol oscillators coupled by resistors or inductors. We can control the synchronization phenomenon by varying the coupling strengths. When three coupling strengths (γ_1, γ_2, γ_3) equal, synchronization phenomena are observed by changing initial value. However when we strengthened up two of γ_1, γ_2 and γ_3, three oscillators of first ring and two oscillators of second ring are synchronized. In the future, we investigate synchronization phenomena using other parameters.