Process of Chaos Propagation in Coupled Chaotic Circuits

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1 Introduction
In this study, we research process of chaos propagation between one chaotic attractor and three-periodic attractors. We observe chaos propagation that three-periodic attractors are affected from the chaotic attractors. Moreover, we investigate convergence time from three-periodic attractors to chaos attractor.

2 System model
Figure 1 shows the chaotic circuit using this study. We propose the single and double ring combination systems as shown in Fig. 2. Central circuit generates the chaotic attractor and the other circuits generate three-periodic attractors in each model. In single model, central circuit is connected to all circuits (see Fig. 2(a)). In double model, central circuit is connected to only 5 circuits and the other 5 circuits are connected with the outside (see Fig. 2(b)).

![Chaotic circuit](image)

Figure: 1  Chaotic circuit.

![Double ring model](image)

Figure: 2  Double Ring

The normalized circuit equations of the system are given as follows:

\[
\begin{align*}
\frac{dx_i}{dt} &= \alpha x_i + z_i \\
\frac{dy_i}{dt} &= z_i - f(y_i) \\
\frac{dz_i}{dt} &= -x_i - \beta y_i - \sum_{j \in S_n} \gamma (z_i - z_j)
\end{align*}
\]

Eq. (1), \( n \) is the number of coupled chaotic circuits up to 11 in this study and \( \gamma \) is the coupling strength. We define \( \alpha_c \) to generate the chaotic attractor and \( \alpha_p \) to generate the three-periodic attractors.

3 Simulation result
3.1 Chaos propagation
Figure 3 shows chaos propagation process in the single ring combination system by increasing the coupling strength. When the coupling strength increase to \( \gamma = 0.0030 \), all three-periodic attractors are propagated to the chaotic attractor (see Fig. 3(c)).

![Chaos propagation (Single ring)](image)

Figure: 3  Chaos propagation (Single ring).

3.2 Convergence time
We investigate convergence time from three-periodic attractors to chaos attractor. The simulation results are summarized in Table 1. Convergence time of single ring model is faster than double ring model. We consider that convergence time is influenced by the number of circuits which is connected with the central circuit.

<table>
<thead>
<tr>
<th>model</th>
<th>single-10</th>
<th>double-5-5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>194654</td>
<td>269522</td>
</tr>
<tr>
<td>2nd</td>
<td>181786</td>
<td>280897</td>
</tr>
<tr>
<td>3rd</td>
<td>219974</td>
<td>289014</td>
</tr>
<tr>
<td>4th</td>
<td>290051</td>
<td>277140</td>
</tr>
<tr>
<td>5th</td>
<td>186565</td>
<td>276105</td>
</tr>
<tr>
<td>Average</td>
<td>196959</td>
<td>277140</td>
</tr>
</tbody>
</table>

4 Conclusion
In this study, we have investigated process of chaos propagation in coupled chaotic circuits. By using computer simulations, we have observed that the chaotic attractor is propagated to the other circuits. The three-periodic attractors are affected from the chaotic attractors when the coupling strength is increased. Furthermore, we have observed that the process to chaos is changed in each model.