Chaos Propagation and Synchronization in Coupled Chaotic Circuits with Star-and-Ring Combination

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***Abstract*—In this study, we investigate chaos propagation and synchronization phenomena in coupled chaotic circuits with star-and-ring combination when one circuit is set to generate chaotic attractor and the other circuits are set to generate three-periodic attractors. Moreover, we observe how to propagate chaos and synchronization phenomena by increasing the coupling strength.**

I. Introduction

Synchronization is one of the fundamental phenomena in nature and one of the typical nonlinear phenomena. Therefore, synchronization of coupled chaotic circuits has been interested by many researchers not only engineering but also the physical and biological fields [1]-[4]. In particular, it is important to investigate synchronization phenomena of coupled circuits under some difficult situations for the circuits. Additionally, it is applicable to the fields of medical science and biology and so on. As previous studies, synchronization and chaos propagation have been investigated in the ring or ladder of coupled chaotic circuits [5]-[7]. However, these studies were considered about the only one ring or ladder system.

In this study, we investigate chaos propagation and synchronization phenomena in coupled chaotic circuits with star-and-ring Combination. We propose a star-and-ring combination system using of 6 chaotic circuits coupled by the resistors. In this model, one circuit is set to generate chaotic attractor and the other circuits are set to generate three-periodic attractors. First, we observe how to chaos propagation by increasing the coupling strength. Moreover, by measuring the phase difference among all adjacent circuits, we investigate synchronization in the entire system.

II. System model

The chaotic circuit is shown in Fig. 1. This circuit consists of a negative resistor, two inductors, a capacitor and dual-directional diodes. We propose a star-and-ring combination system model in Fig. 2. In this system, 6th circuit generates chaotic attractor and the other circuits generate three-periodic attractors. In order to investigate the chaos propagation, we use two parameters of the coupling strength. Each circuit is coupled by the resistor.

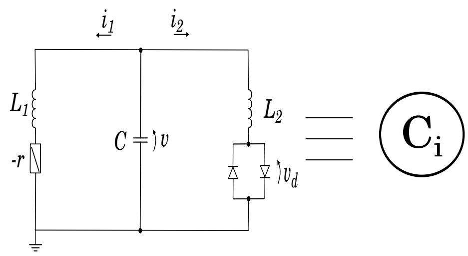

Fig. 1. Chaotic circuit.

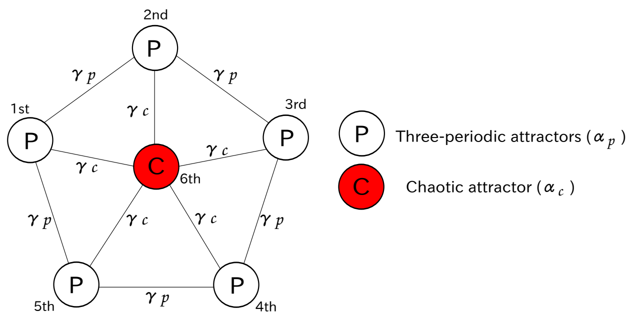

Fig. 2. Star-and-ring combination.

The circuit equations of this circuit are described as follows:

$$\begin{cases} L_1 \dfrac{di}{dt} = v + ri \\ L_2 \dfrac{di}{dt} = v - v_d \\ C \dfrac{dv}{dt} = -i_1 - i_2, \end{cases} \qquad (1)$$

The characteristic of nonlinear resistance is described as follows:

$$v_d = \frac{r_d}{2} \left(\left| i_2 + \frac{V}{r_d} \right| - \left| i_2 - \frac{V}{r_d} \right| \right). \qquad (2)$$

By changing the variables and parameters,

$$\begin{cases} i_1 = \sqrt{\dfrac{C}{L_1}} V x_n, \quad i_2 = \dfrac{\sqrt{L_1 C}}{L_2} V y_n, \quad v = V z_n \\ \alpha = r\sqrt{\dfrac{C}{L_1}}, \quad \beta = \dfrac{L_1}{L_2}, \quad \delta = r_d \dfrac{\sqrt{L_1 C}}{L_2}, \\ \gamma = \dfrac{1}{R}\sqrt{\dfrac{L_1}{C}}, \quad t = \sqrt{L_1 C_2}\tau, \end{cases} \tag{3}$$

The normalized circuit equations are given as follows:

$$\begin{cases} \dfrac{dx_n}{d\tau} = \alpha x_n + z_n \\ \dfrac{dy_n}{d\tau} = z_n - f(y_n) \\ \dfrac{dz_n}{d\tau} = -x_n - \beta y_n - \displaystyle\sum_{n=1}^{N} \gamma_{ij}(z_n - z_{n+1}) \\ \qquad\qquad (n = 1, 2, \cdots, N). \end{cases} \tag{4}$$

In Eq. (4), N is the number of coupled chaotic circuits and γ is the coupling strength. $f(y_i)$ is described as follows:

$$f(y_n) = \frac{1}{2}\left(\left|y_n + \frac{1}{\delta}\right| - \left|y_n - \frac{1}{\delta}\right|\right). \tag{5}$$

The coupling strength which connected with 6th is γ_c, and the others are γ_p. We define α_c to generate the chaotic attractor, and α_p is defined to generate the three-periodic attractors. For the computer simulations, we calculate Eq. (4) using the fourth-order Runge-Kutta method with the step size h = 0.01. In this study, we set the parameters of the system as $\alpha_c = 0.460$, $\alpha_p = 0.412$, $\beta = 3.0$ and $\delta = 470.0$.

III. Simulation Results

A. Chaos propagation

Figure 3 shows the initial state when all circuits are not connected. We observe three-periodic attractors from 1st to 5th circuits and chaotic attractor in 6th circuit.

Figure 4 shows chaos propagation attractors when the coupling strength γ_c is only conected from 6th to 5th circuit. At this time, we fix the coupling strength as $\gamma_c = 0.0007$, and we increase coupling strength γ_p. The 5th circuit is only propagated the chaotic attractor of 6th chaotic circuit (see. Fig. 4(a)). The chaotic attractor of 5th circuit propagates to both side of the neighbor circuits (see. Fig. 4(b)). The states of all circuits become the chaotic attractors (see. Fig. (c)).

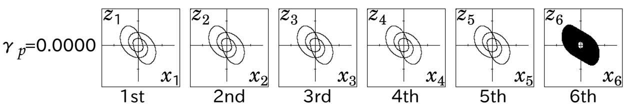

Fig. 3. Chaos propagation ($\gamma_c = 0.0000$).

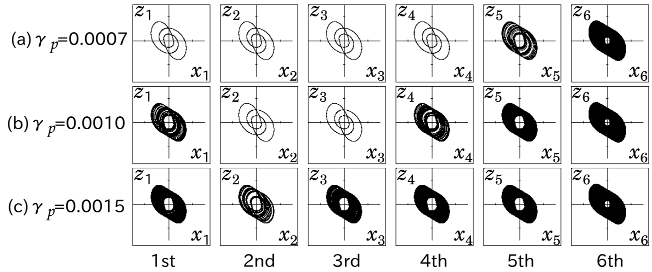

Fig. 4. Chaos propagation ($\gamma_c = 0.0007$).

Figure 5 shows the relation between the phase difference and the coupling strength when the coupling strength γ_c is only conected from 6th to 5th circuit. At this time, we fix the coupling strength as $\gamma_c = 0.0007$, and we increase coupling strength γ_p. The phase difference shows the average among all adjacence circuits. If all circuits are not synchronized, the phase difference shows 60°. We can confirm that the phase difference is smaller and comes close to 0° by increasing the coupling strength.

As a result, three-periodic attractors are affected from chaotic attractor by increasing the coupling strength γ_p.

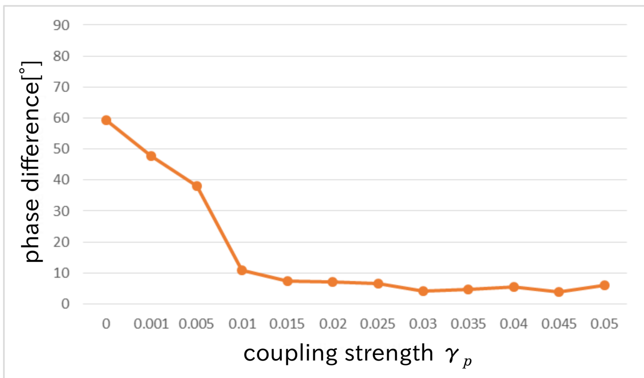

Fig. 5. Relation between the phase difference and the coupling strength in star-and-ring combination.

B. Synchronization phenomena

In this section, we confirm the variation of the phase difference, when we change the number of edges between 6th circuit and the other circuits. Figure 6 shows all system patterns in this star-and-ring combination by changing combination. For example, this model is connected to all circuits (see. Fig. 6(g)) and this model is only connected from 6th to 5th circuits (see. Fig. 6(a)). When we fix the coupling strength as $\gamma_c = 0.007$ and $\gamma_p = 0.005$, the phase difference shows the average among all adjacence circuits in Table 1.

In order to investigate the phase difference, we consider that the entire system easy to reach the synchronous state by increasing the number of edge.

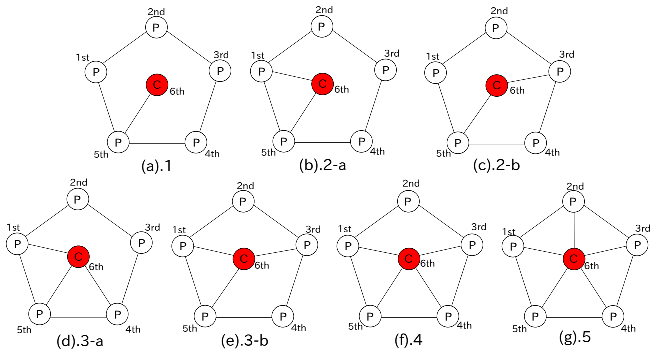

Fig. 6. System patterns.

TABLE I
PHASE DIFFERENCES IN ALL SYSTEMS.

patterns	phase difference[°]	average phase difference[°]
(a).1	56.11	56.11
(b).2-a	18.54	25.57
(c).2-b	32.59	
(d).3-a	37.98	29.79
(e).3-b	21.60	
(f).4	11.85	11.85
(g).5	9.06	9.06

IV. CONCLUSIONS

In this study, we have investigated chaos propagation and synchronization phenomena in coupled chaotic circuits as our proposed system. By the computer simulations, We have observed that the chaotic attractor is propagated to the other circuits. The three-periodic attractors are affected from the chaotic attractors when the coupling strength increase. Moreover we consider that the phase difference close synchronous state by increasing the number of edge.

For the future works, we develop the network model into cubic and more complex. Considering the other network of chaotic circuits is important subjects for us.

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