Synchronization in Small-World Chaotic Circuit Network under Parameter Mismatch

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***Abstract*—In this study, we investigate the synchronization of coupled chaotic circuits in the presence of parameter mismatch. We consider three different network topologies obtained from the small-world network model and two parameter dispersions of mismatched circuits. By means of the computer calculations, the synchronization probabilities of the entire network are investigated during a certain time interval. From the simulation results, we find that the small-world network provides the best framework to realize synchronization.**

I. Introduction

Complex networks have attracted a great deal of attention from various fields since the discovery of "small-world" network [1] and "scale-free" network [2]. In particular, how network topological structure influences its dynamical behaviors, is a significant hot topic. In order to apply for practical applications in many disciplines, understanding the relationship between topological structure and functional behavior on the networks can be considered as an important problem. As the dynamics on the networks, the synchronization is one of the fundamental phenomenon in various fields. Especially, chaotic synchronization is very interesting phenomenon, have received a great deal of attention since the report by Pecora and Carrol [3]. However, there are not many studies for complex networks of continuous-time real physical systems such as electrical circuits. In our previous work, we have investigated the synchronization phenomena of coupled chaotic circuits on a complex network with local bridge [4]. We have focused on local bridge structure observed from the small-world network. However, the circuit parameters were fixed with same parameters for all chaotic circuits and the only one network model was considered.

In this study, we investigate the global synchronization of coupled chaotic circuits in the small-world network. Wan and Chen reported the synchronization in the small-world coupled Chua's circuits [5]. We focus on the parameter mismatches, the synchronization of coupled chaotic circuits in three different network topologies obtained from the small-world network model is studied. From the simulation results, synchronization probability during a certain time interval are compared among three network topologies. Thereby, the small-world topology is shown to be effective for the achievement of the synchronization in the entire network.[1]

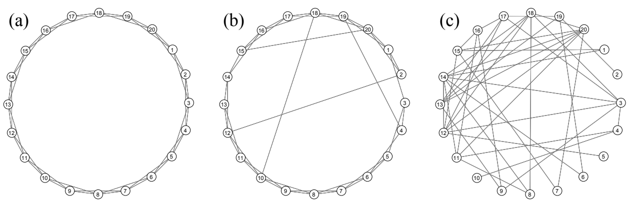


Fig. 1. Illustration of the WS model for $N = 20$ and $k = 4$. (a) : Regular network, $p = 0$. (b) : Small-world network, $p = 0.1$. (c) : Random network, $p = 1$.

II. Small-World Network Model

In 1998, Watts and Strogatz introduced very interesting small-world network model, called the WS model [1]. The WS model can be generated as shown in Fig. 1. Starting from a ring lattice with N nodes and k edges per nodes in Fig. 1(a), each edge is rewired at randomly with probability p. By increasing the probability p, the randomness of the network are also increased. The small-world network is known as the graph which is characterized by highly clustering coefficient like a regular graph and small path length like a random graph.

Topological structures in complex networks of N nodes and E edges can be evaluated by the typical three structural metrics (degree, clustering coefficient and path length). First, degree (k) shows the number of edges on a node. Second, clustering coefficient (C) shows the number of actual links between neighbors of a node divided by the number of possible links between those neighbors. This is given as follows:

$$C = \frac{1}{N}\sum_{n=1}^{N} C_n = \frac{1}{N}\sum_{n=1}^{N}\frac{2E_n}{k_n(k_n-1)}. \quad (1)$$

[1]We have already presented this result in Proc. of NOLTA'15, pp. 431-444, Dec. 2015.

TABLE I
PROPERTIES OF THREE NETWORKS AS SHOWN IN FIG. 1.

	Regular	Small-world	Random
p	0	0.1	1
C	0.500	0.358	0.216
L	2.895	2.458	2.221

Third, path length (L) shows the shortest path in the network between two nodes. This is given as follows:

$$L = \frac{2}{N(N-1)} \sum_{m=1}^{N-1} \sum_{n=m+1}^{N} l(m,n). \tag{2}$$

In this research, we consider coupled chaotic circuits in three different network topologies obtained from the WS model in Fig. 1. Each topologies is called regular, small-world and random networks, respectively. Table 1 shows the properties of three networks as shown in Fig. 1.

III. Coupled Chaotic Circuit

Figure 2 shows the chaotic circuit which is three-dimensional autonomous circuit proposed by Shinriki *et al.* [6][7]. This circuit is composed by an inductor, a negative resistor, two capacitors, and dual-directional diodes. In this study, we propose 20 coupled chaotic circuits in three network topologies as shown in Fig. 1. In these network models, chaotic circuits are applied to each node of the networks and each edge corresponds to a coupling resistor R.

First, the circuit equations are given as follows:

$$\begin{cases} L\dfrac{di_n}{dt} &= v_{2n} \\ C_1\dfrac{dv_{1n}}{dt} &= gv_{1n} - i_{dn} - \dfrac{1}{R}\displaystyle\sum_{k\in S_n}(v_{1n}-v_{1k}) \\ C_2\dfrac{dv_{2n}}{dt} &= -i_n + i_{dn}, \end{cases} \tag{3}$$

where $n = 1, 2, 3, ..., 20$ and S_n is the set of nodes which are directly connected to the node n. We approximate the $i-v$ characteristics of the nonlinear resistor consisting of the diodes by the following three-segment piecewise-linear function:

$$i_{dn} = \begin{cases} G_d(v_{1n} - v_{2n} - V) & (v_{1n} - v_{2n} > V) \\ 0 & (|v_{1n} - v_{2n}| \leq V) \\ G_d(v_{1n} - v_{2n} + V) & (v_{1n} - v_{2n} < -V). \end{cases} \tag{4}$$

By using the parameters and the variables:

$$\begin{cases} i_n = \sqrt{\dfrac{C_2}{L}}Vx_n,\ v_{1n} = Vy_n,\ v_{2n} = Vz_n \\ t = \sqrt{LC_2}\tau,\ \text{“}\cdot\text{”} = \dfrac{d}{d\tau},\ \alpha = \dfrac{C_2}{C_1} \\ \beta = \sqrt{\dfrac{L}{C_2}}G_d,\ \gamma = \sqrt{\dfrac{L}{C_2}}g,\ \delta = \dfrac{1}{R}\sqrt{\dfrac{L}{C_2}}, \end{cases} \tag{5}$$

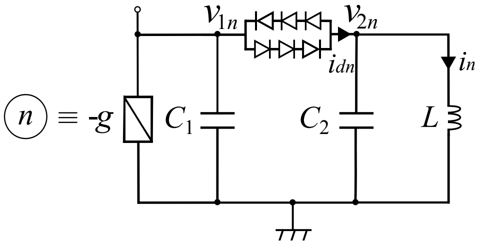


Fig. 2. Chaotic circuit.

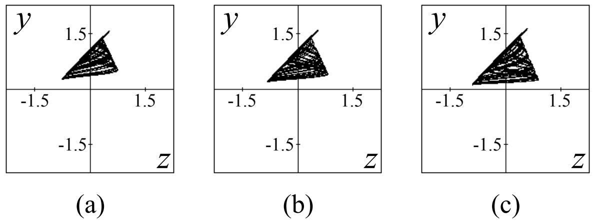


Fig. 3. Chaotic attractor of the circuit as shown in Fig. 2. $\beta = 20$, $\gamma = 0.5$. (a) : $\alpha = 0.46$, (b) : $\alpha = 0.50$, (c) : $\alpha = 0.54$,

the normalized circuit equations are given as follows:

$$\begin{cases} \dot{x_n} &= z_n \\ \dot{y_n} &= \alpha\gamma y_n - \alpha f(y_n - z_n) - \alpha\delta\displaystyle\sum_{k\in S_n}(y_n - y_k) \\ \dot{z_n} &= f(y_n - z_n) - x_n. \end{cases} \tag{6}$$

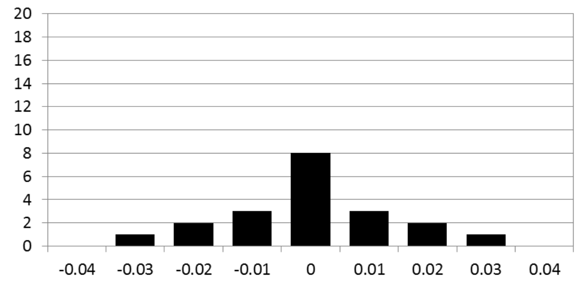
The nonlinear function $f(y_n - z_n)$ corresponds to the $i-v$ characteristics of the nonlinear resistor consisting of the diodes and are described as follows:

$$f(y_n - z_n) = \begin{cases} \beta(y_n - z_n - 1) & (y_n - z_n > 1) \\ 0 & (|y_n - z_n| \leq 1) \\ \beta(y_n - z_n + 1) & (y_n - z_n < -1). \end{cases} \tag{7}$$

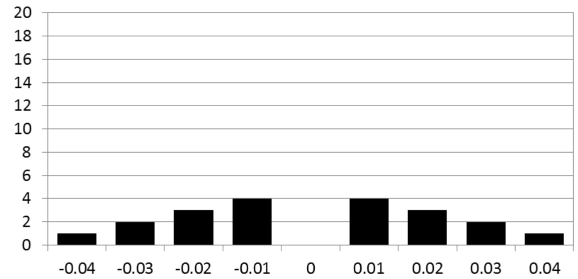
This circuit generates asymmetric chaotic attractor as shown in Fig. 3. The values y and z in Fig. 3 correspond to v_1 and v_2 of the circuit in Fig. 2, respectively. By increasing parameter α, the range of chaotic trajectory becomes widely formed.

IV. Parameter Dispersion

In this research, we fix the circuit parameters as $\alpha = 0.50$, $\beta = 20$, $\gamma = 0.50$ and $\delta = 0.70$ for all chaotic circuits. Additionally, the parameter mismatches $\Delta\alpha$ are added for each circuit parameter α which relate to the chaos degree. Namely, the parameter α of each circuit is shown as $\alpha = 0.5 + \Delta\alpha$, respectively. We propose the parameter dispersion as shown in Fig. 4. Each pattern is different in the number of the parameter mismatched circuits and the range of the parameter mismatches. By using the proposed parameter dispersion patterns, we add the parameter mismatches for the circuits in the computer simulations.

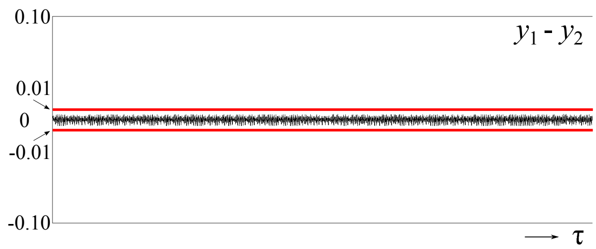


(a) Pattern 1.

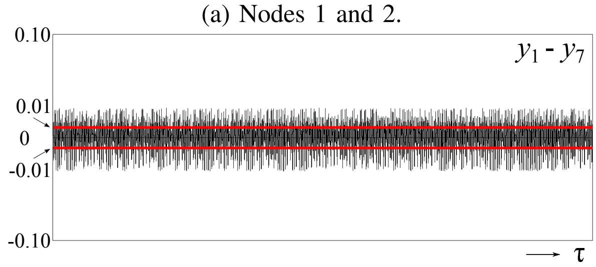


(b) Pattern 2.

Fig. 4. Proposed two patterns of the parameter dispersion. Vertical axis: the number of circuits. Horizontal axis: parameter mismatches $\Delta\alpha$.



(a) Nodes 1 and 2.



(b) Nodes 1 and 7.

Fig. 5. The example of the voltage difference between two circuits when the parameter dispersion pattern 2 in the small-world network and the definition of the synchronization. (a) : Synchronization. (b) : Asynchronization.

V. Synchronization

A. Definition of Synchronization

Figure 5 shows the example of the computer simulation results when the parameter dispersion pattern 2 (see Fig. 4(b)) in the small-world network. The vertical axes are the difference between the voltages (corresponding to v_1 of the circuit in Fig. 2) of the nodes 1 and 2 or 7. Namely, if the two nodes are synchronized, the value of the graph should be almost zero. In order to analyze the synchronization state, we define the synchronization by the following equation:

$$|y_i - y_j| < 0.01 \quad (i \neq j). \tag{8}$$

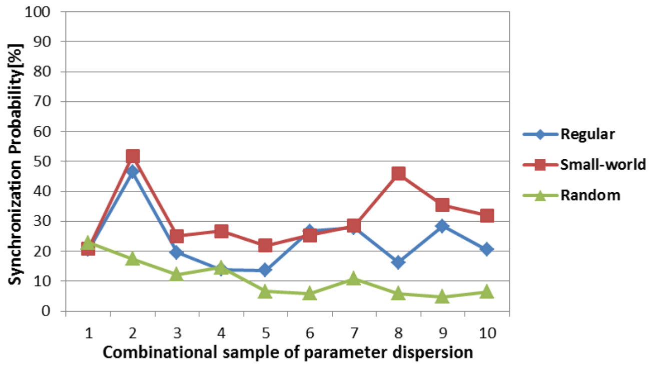
By means of the above definition of the synchronization, we define that the nodes 1 and 2 in Fig. 5(a) are synchronized perfectly. However, the nodes 1 and 7 in Fig. 5(b) are almost evaluated as the asynchronization in this definition. Thus, we propose and investigate the synchronization probability denoted the synchronization rate during a certain time interval. In this research, we fix a certain time interval as ($\tau = 1,000,000$ and $step = 0.01\tau$) and statistically investigate the synchronization probability in the entire network of 20 coupled chaotic circuits.

B. Synchronization Probability

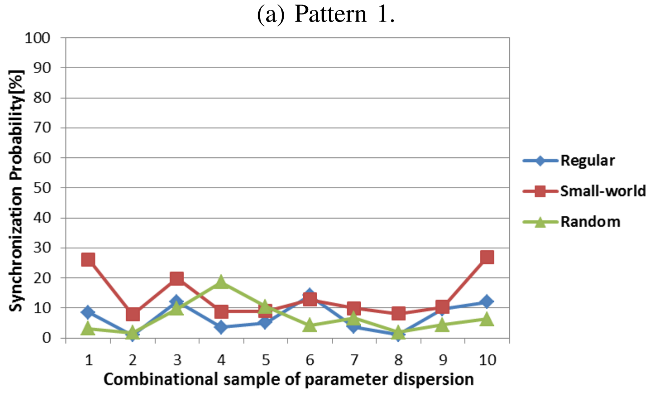
Figure 6 shows the investigation results of the relationship between the synchronization probability and the combinational samples of two parameter dispersion patterns in three network topologies. Each pattern corresponds to the two patterns in Fig. 4. The vertical axes denote the synchronization probability in the entire network during the time interval. The horizontal axes denote the combinational samples considered from each parameter dispersion pattern. The combinational of the parameter dispersion is considered a large number. Therefore, we choose the 10 samples randomly as the combinational of the parameter dispersion in each pattern of Fig. 4. From Fig. 6, we confirm that the networks become to be difficult for the global synchronization by increasing the number of the parameter mismatched circuits. On the other hand, the synchronization probability in each pattern depends on the combinational sample. This reason is that $N = 20$ would be too small. However, we found that the synchronization probability tends to be higher in the small-world network. Therefore, we consider that the small-world topology is effective for the synchronization in the entire network. In addition, the complex relation between global synchronization of the location of mismatched circuit can be confirmed in this research. More detailed these relation considering more large number of samples should be investigated for our future works.

VI. Conclusion

This paper considered the synchronization of coupled chaotic circuits with the parameter mismatches in three network topologies obtained from the WS model. In particular, we focused on the parameter distribution based on the number of parameter mismatched circuits. By means of the computer calculations, the synchronization probabilities of the entire network were investigated during a certain time interval. From the simulation results, we found that the synchronization probability tends to be higher in the small-world network. We consider that the small-world network enhances synchronization. However, the fluctuation is large among the combinational samples. This reason is that $N = 20$ would be too small. Therefore, more detailed investigation considering more large-scale networks should be carried out in our future works.



(a) Pattern 1.



(b) Pattern 2.

Fig. 6. Relationship between the synchronization probability and the combinational sample in three network topologies.

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