

# Synchronization Phenomena in Star Combination of van der Pol Oscillators with Different Frequencies

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## 1. Introduction

Synchronization phenomena can be described in the following fields, engineering, physics and so on. In this study, we investigate the effect to three-coupled central star circuits by adding another oscillator with different frequency.

## 2. Circuit model

The circuit model used in this study is shown in Fig. 1. Three van der Pol oscillators connected as the star combination. In addition, we also add the 4th van der Pol oscillator on the three-couple star circuits. So we change the frequency of 4th oscillator and investigate the influence of 4th oscillator for the overall star circuit.

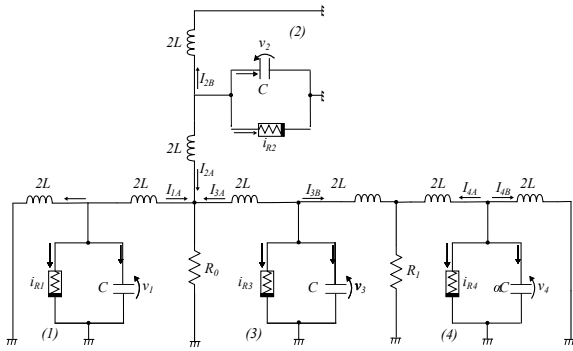


Figure 1: Circuit model.

The normalized equations are represented as follows:

$$\begin{cases} \frac{dx_k}{d\tau} = \varepsilon(x_k - \frac{1}{3}x_k^3) - y_k - z_k & (k = 1, 2, 3) \\ \frac{dy_k}{d\tau} = \frac{1}{2}x_k - \frac{1}{2}\beta_0(y_1 + y_2 + y_3) & (k = 1, 2, 3) \\ \frac{dz_k}{d\tau} = \frac{1}{2}x_k & (k = 1, 2, 4) \\ \frac{dz_3}{d\tau} = \frac{1}{2}x_3 - \beta_1(z_3 + y_4) \\ \frac{dx_4}{d\tau} = \omega^2(\varepsilon(x_4 - \frac{1}{3}x_4^3) - y_4 - z_4) \\ \frac{dy_4}{d\tau} = \frac{1}{2}x_4 - \frac{1}{2}\beta_1(z_3 + y_4) \end{cases}$$

where  $\varepsilon$  is the non-linear intensity.

## 3. Simulation Results

We investigate synchronization phenomena and oscillation of the oscillators by using computer simulation with  $\beta_0=0.1$  and  $\beta_1=0.3$ . We investigate the change of varying  $\omega$  ( $\omega=1$  to  $\omega=1.8$ ). Figures 2, 3 and 4 show the simulation results.

In Fig. 2, in the case of  $\omega = 1$ , all the four oscillators oscillated. Only between 4th oscillator and 1st oscillator are synchronized at anti-phase. Consequently, we did not

see the effects of omega to star circuit. And then, as the  $\omega$  increases to 1.2, the oscillation of 4th oscillator and the oscillation of 3rd oscillator stop, namely oscillation death appears as shown in Fig. 3. At the same time, between the 1st oscillator and the 2nd oscillator become anti-phase synchronization. From here we can see effect of  $\alpha$  to star circuit.

In Fig. 4, as  $\omega$  increases from 1.2 to 1.8, 1st oscillator and 2nd oscillator still are anti-phase synchronization, however the fourth oscillator oscillates again and frequency of the 4th oscillator become faster than before from above 1.8 and higher .

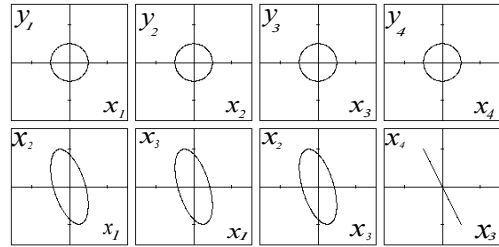


Figure 2: Simulation result ( $\omega=1$ ).

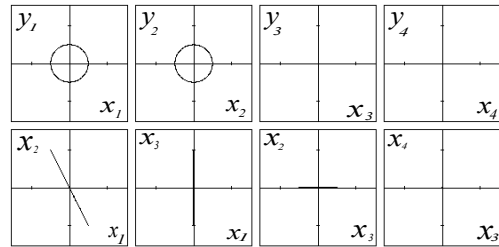


Figure 3: Simulation result ( $\omega=1.2$ ).

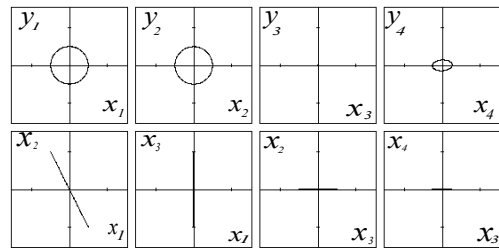


Figure 4: Simulation result ( $\omega=1.8$ ).

## 4. Conclusions

In this study, we have investigated synchronization phenomena and oscillation of four oscillators with different frequencies. By carrying out computer simulations, we confirm that oscillation of the 3th and 4th oscillator stop for  $\omega=1.2$ .