Synchronization Changing Local Coupling Strength with Hindmarsh Rose Model

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1. Introduction
In recent years, there has been considerable interest in building connections of neurons in a brain. The coupling strengths between neurons become strong if the neurons communicate with each other. The neurons have the axon. Some axons are covered with the oligodendrocyte. The covered axons communicate quicker than the non-covered ones. This phenomenon is called myelination. The myelination has big relation to learning. Therefore, the study about myelination has possibility to make more performance for learning by adapting application. In this study, we investigate synchronization by changing local coupling strengths between neurons.

2. System model
In this study, we use Hindmarsh-Rose (HR) model. This model has three variables. These are the membrane potential $z(t)$, auxiliary variable called $y(t)$ representing a set of fast ion channels connected with aspects of potassium and sodium transport, and a 'slow' variable $z(t)$ which captures the slower dynamics of yet other ion channels. The HR model is written as the following equations.

$$\begin{align*}
\dot{x}_i(t) &= y_i(t) - ax_i^3(t) + bx_i^2(t) - z_i(t) + I - K_i \\
\dot{y}_i(t) &= c - d_2z_i^2(t) - y_i(t) \\
\dot{z}_i(t) &= -rz_i(t) + rS(x_i(t) - c_i)
\end{align*}$$

where $K_i$ is given as:

$$\begin{align*}
K_1 &= \varepsilon_1(x_1 - x_2) \\
K_2 &= \varepsilon_1(x_2 - x_1) + \varepsilon_2(x_2 - x_3) \\
K_3 &= \varepsilon_2(x_3 - x_2) + \varepsilon_3(x_3 - x_4) \\
K_4 &= \varepsilon_3(x_4 - x_3)
\end{align*}$$

where the parameter $i$ is chosen by 1, 2, 3 and 4. The parameters are given as $a = 1$, $b = 3$, $c = 1$ and $d = 5$. We use the injected current $I = 3.281$, the voltage $c_i = -1.6$, the scale of the influence of the membrane voltage on the slow dynamics $S = 4.0$ and the time scale for the slow adaptation current $r = 0.0021$. The differential equations are solved by the Runge-Kutta method.

3. Simulation results
The research of two neurons using HR model has already obtained results[1]. In this simulation, we use the ladder of four neurons which are coupled by electrical coupling. Figure 1 illustrates in the ladder model. Figure 2 shows the temporal evolution of the membrane potential in neuron 1, 2, 3 and 4. Figure 2 (a), (b), (c) and (d) indicates the membrane potential in the neuron 1, 2, 3 and 4. When the coupling strengths are fixed as $\varepsilon_1 = 0.58$, $\varepsilon_2 = 0.28$ and $\varepsilon_3 = 0.58$, we obtain results. The neuron 1 is in phase with neuron 2, however it is the anti phase with the neuron 3 and 4. Figure 3 shows lissajous figures about in phase and anti phase. The neuron 1, 2 and 3, 4 indicate Fig. 3 (a) and the others indicate Fig. 3 (b).

Figure 1: The ladder of four neurons on HR model.

Figure 2: The temporal evolution of the membrane potential in each neuron. (a) The membrane of neuron 1. (b) The membrane of neuron 2. (c) The membrane of neuron 3. (d) The membrane of neuron 4.

Figure 3: The lissajou figure. (a) In phase. (b) Anti phase.

4. Conclusions
In this study, we have investigated the synchronization with the ladder of four neurons in HR model. The coupling strengths are fixed with different values. The simulation results indicate in phase and anti phase synchronization. In our future works, we observe synchronization by other coupling strengths. Furthermore, we would like to do the simulation with large scale ladder model.

Reference