Synchronization Phenomena of Four Neurons on FitzHugh-Nagumo Model

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1. Introduction

Some neural oscillator models are studied by many researchers. FitzHugh-Nagumo (FHN) model is one of these models. Periodic solutions on a pair of FHN models have already found[1]. In addition, the periodic solutions have some ring patterns. We propose a system which a neuron is connected to a triangle system. The system has four neurons with inhibitory coupling. We focus on synchronization phenomena of the system. Likewise, we investigate ring patterns.

2. FitzHugh-Nagumo model

In the following, we study the system which is modeled by the excitable FHN models. FHN model is given by the following equations:

\[ \frac{du_i}{dt} = u_i(u_i - \alpha)(1 - u_i) - v_i + \frac{K}{N} \sum_{i\neq j} (u_j - u_i) \]  \hspace{1cm} (1)

\[ \frac{dv_i}{dt} = \tau (u_i - \gamma v_i) \] \hspace{1cm} (2)

where \( u_{ij} \) is the activator, \( v_{ij} \) is the inhibitor, \( \alpha, \tau \) and \( \gamma \) are parameters, \( K \) is the coupling strength and \( N \) is the number of elements. \( \alpha, \tau \) and \( \gamma \) are fixed at \( \alpha = 0.01, \tau = 0.001 \) and \( \gamma = 0.0 \) because these values are used in previous study of a pair of excitable FHN elements.

3. Simulation results

The simulation model is shown in Fig. 1. We investigate synchronization phenomena by changing the coupling strength \( K \) from \(-1\) to \(0\).

As the results of simulation, in case of \( K = -0.035 \), the observed time waveforms are shown in Fig. 2. Time waveforms \( N_1 \) and \( N_3 \) are synchronized at in-phase with \( N_2 \) and \( N_4 \). Next, we explain two-phase firing pattern in detail. After \( N_1 \) and \( N_3 \) excite, \( N_2 \) and \( N_4 \) excite soon. After these successive excitations, all neurons stay quiescent state for a while. On the next successive excitations, \( N_2 \) and \( N_4 \) excite first. After that, \( N_1 \) and \( N_3 \) excite soon. These states are repeated.

![Figure 1: The system of four neurons on FHN model.](image)

![Figure 2: Time evolution of four neurons on FHN model for the coupling strength \( K = -0.035 \). The solid lines of (a) and (b) correspond to time waveforms of \( N_1(N_3) \) and \( N_2(N_4) \), respectively. The solid lines of (c) correspond to all neuronal time waveforms.](image)

4. Conclusions

In this study, we proposed a system which a neuron is connected to a triangle system. Periodic solution was observed on the system. Furthermore, we found a simple firing pattern. These facts shed some light on neural activity in the brain and the spinal cord where periodic behavior is often observed.

5. Reference