**1-4**

# Synchronization and Chaos Propagation between Chaotic and Periodic Attractors in Symmetric System

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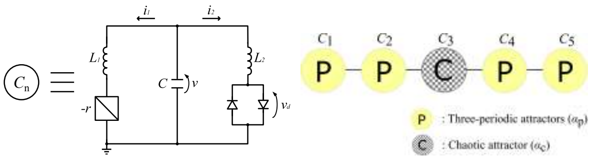
## 1. Introduction

In this study, we investigate synchronization and chaos propagation in five coupled chaotic circuits in various systems. We propose a ladder system that the central circuit generates chaotic attractor and the other circuits generate three-periodic attractors. Moreover, we compare the phase difference among symmetric systems in the cases of adding the coupling resistor from the ladder system.

## 2. System model

Figure 1(a) shows the chaotic circuit. we propose the ladder system model in Fig. 1(b). Each circuit is coupled by the resistors which correspond to the edges in this system. The central circuit generates the chaotic attractor and the other circuits generate the three-periodic attractors.




(a) Chaotic circuit. (b) Ladder system model.

Figure 1: System model.

In the proposed ladder system, the circuits are connected to only adjacent circuits by the resistors. The normalized circuit equations of the system are given as follows:

$$\begin{cases} \dot{x_n} = \alpha x_n + z_n \\ \dot{y_n} = z_n - \dfrac{\delta}{2}\left(\left|y + \dfrac{1}{\delta}\right| - \left|y - \dfrac{1}{\delta}\right|\right) \\ \dot{z_n} = -x_n - \beta y_n - \displaystyle\sum_{m \in s_n} g(z_n - z_m), \end{cases} \quad (1)$$

where $\alpha$ represents the chaos degree. $n$ represents the circuit number up to 5 in this study. $S_n$ is the set of circuits which are directly connected to $C_n$. $g$ represents the coupling strength corresponding the coupling resistor $R$.

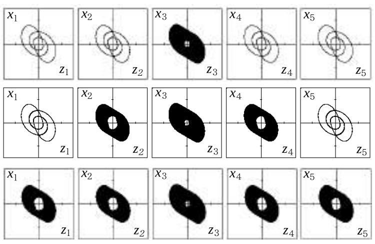



Figure 2: Attractor in the laddr system.

## 3. Simulation results

Figure 2 shows some examples of the computer simulation results. Figure 2(a) shows the initial state when all circuits are not connected. We observe the state that the chaos are propagated to the only adjacent circuits from the central chaotic circuit in a range of the coupling strength $g$ (see Fig. 2(b)). By increasing the coupling strength $g$, all three-periodic attractors are affected from chaotic attractor and all circuits close to the synchronous state(see Fig. 2(c)). Moreover, Fig. 3 shows the four symmetric systems considered by adding three edges to the ladder system of Fig. 1(b).

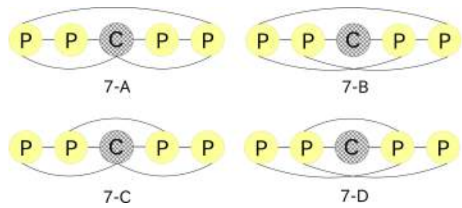



Figure 3: Four symmetric systems.

Table 1 shows the phase differences of ladder system and the four symmetric systems in the case of $g = 0.01$ and each system pattern corresponds to Fig. 3. In Table 1, the phase differences are masured between the adjacent circuits and the phase differences shows the average in each entire system. From Table 1, the phase differences in the four symmetric systems are smaller than the ladder system. Additionally, the phase differences of 7-A and 7-C are smaller than 7-B and 7-D in the four symmetric systems. This reason can be considered that the central chaotic circuit is connected to all circuits in the systems of 7-A and 7-C.

Table 1: Phase differences in the symmetric systems ($g = 0.01$).

| edges | phase difference [° ] | | | | |
|---|---|---|---|---|---|
| | $C_1$-$C_2$ | $C_2$-$C_3$ | $C_3$-$C_4$ | $C_4$-$C_5$ | Average |
| 4 (Ladder) | 13.08 | 16.91 | 23.41 | 12.38 | 16.45 |
| 7-A | 7.14 | 12.12 | 7.49 | 6.51 | 8.32 |
| 7-B | 6.65 | 22.48 | 6.66 | 6.46 | 10.57 |
| 7-C | 7.51 | 11.09 | 7.88 | 7.96 | 8.61 |
| 7-D | 6.77 | 19.73 | 6.79 | 7.65 | 10.24 |

## 4. Conclusions

In this study, we have researched about synchronization and chaos propagation of five coupled chaotic circuits in the proposed systems. We observed that the chaotic attractor of the central circuit propagates to all circuits by increasing the coupling strength in the ladder system. Moreover, we compared the phase difference among the ladder system and the four symmetric systems. As a result, the entire system is easy to reach a state of synchronization when the central chaotic circuit is connected to many circuits.