Propagation of Switching Phenomena observed on a Ring of Coupled Chaotic Circuits in State of *N*-Phase Synchronization

Takuya Nishimoto*, Yoko Uwate*, Yasuteru Hosokawa†, Yoshifumi Nishio* and Daniéle Fournier-Prunaret‡
*Dept. of Electrical and Electronic Eng., Tokushima University,
2-1 Minami-Josanjima, Tokushima, 770-8506 Japan
Email: {nishimoto, uwate, nishio}@ee.tokushima-u.ac.jp
†Dept. of Media and Information Systems, Shikoku University,
Furukawa, Ohjin, Tokushima, 771-1192, Japan
Email: hosokawa@keiei.shikoku-u.ac.jp
‡LAAS-CNRS, 7 avenue du colonel Roche, F-31077 Toulouse Cedex 4, France
Email: daniele.fournier@insa-toulouse.fr

***Abstract*—2 types of synchronization phenomena, which are in-phase and N-phase synchronization, can be observed on a ring of coupled chaotic circuits. In state of N-phase synchronization, the system generates propagation of switching phenomena. In this study, we focus on the relationships between the propagation of switching phenomena and parameters. Various propagation patterns depending on the number of circuits and propagation velocity are investigated.**

I. INTRODUCTION

Chaos synchronization observed on large-scale coupled chaotic systems have been attracted attention in various science fields because it can be regarded as models of a lot of real physical systems. For instance, chaos phenomena have been reported in engineering, biology, economics, astronomy and so on. Therefore the investigation of synchronization phenomena on high-order chaotic systems are very important to grasp essentials of the phenomena observed in natural system.

On the other hand, some chaotic circuit have coexisting attractors [1]. Multi-scroll attractor observed on coupled chaotic system causes some interesting phenomena [2], [3]. In our past study, we investigated coexisting synchronization phenomena on a ring of coupled chaotic system [4]. The phenomena are interesting since the system generates N-phase synchronization phenomena although the coupling element is simple positive resister. Moreover, we analyzed the phenomena by carrying out averaging method on a ring of coupled van der Pol circuit and clarified the stable parameter region of N-phase synchronization.

In this study, we investigate propagation of switching phenomena observed on the ring of coupled chaotic system in state of N-phase synchronization. In the past study, asynchronous double-mode oscillations and wave propagations have reported [5], [6]. However, observation of propagation phenomena on a coupled chaotic circuit coupled by simple positive resister have not been reported before. Thus, propagation of switching phenomena in state of N-phase synchronization on coupled chaotic circuit coupled by simple positive resister is the first observation. Various propagation patterns and relationships between propagation velocity and parameters are investigated in proposed system model. By increasing the number of circuits, propagation patterns are increased and it means a lot of coexisting state can be observed.

This paper is organized as follows. In Section II, we propose a ring of coupled chaotic system. In Section III, 2-type chaos synchronization are shown. Be based on Section III, propagation of switching phenomena and its patterns are shown in Section IV. Finally, Section V concludes this paper.

II. SYSTEM MODEL

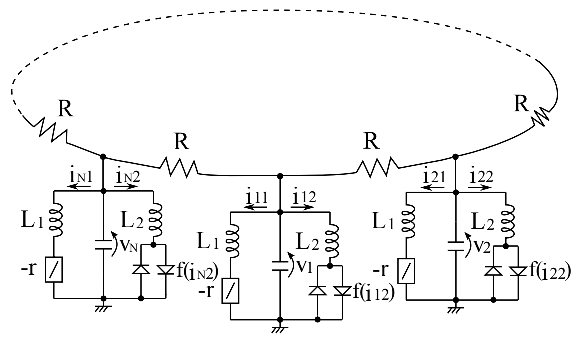


Fig. 1. System model.

The system model is shown in Fig. 1. Each subcircuit is coupled by resistor. Subcircuit is chaotic circuit which consists of three memory elements, one linear negative resistor, and bi-directionally-coupled diodes proposed by Nishio et al [8]. By changing the parameters and variables,

$$\begin{aligned}
&t = \sqrt{L_1 C}\tau, i_{n1} = V\sqrt{\frac{C}{L_1}}x_n, i_{n2} = V\sqrt{\frac{C}{L_1}}y_n,\\
&v_n = Vz_n, \text{"}\cdot\text{"} = \frac{d}{d\tau}, \alpha = r\sqrt{\frac{C}{L_1}}, \beta = \frac{L_1}{L_2} \qquad (1)\\
&\gamma = r_d\sqrt{\frac{C}{L_1}} \text{ and } \delta = \frac{1}{R}\sqrt{\frac{L_1}{C}}
\end{aligned}$$

the normalized circuit equation is described as following $3\times N$-dimensional ordinary differential equation:

$$\left\{\begin{array}{rcl}
\dot{x}_n &=& \alpha x_n + z_n,\\
\dot{y}_n &=& \beta\left\{z_n - \hat{f}(y_n)\right\},\\
\dot{z}_n &=& -x_n - y_n\\
&& +\delta\left(z_{n-1} - 2z_n + z_{n+1}\right),\\
&& \qquad (n = 1, 2, \cdots, N)
\end{array}\right. \qquad (2)$$

where

$z_0 = z_N, \quad z_{N+1} = z_1$ and

$$\hat{f}(y_n) = \tfrac{1}{2}\left(|\gamma y_n + 1| - |\gamma y_n - 1|\right).$$

Figure 2 shows an example of asymmetric chaos attractor depending on initial value obtained from one subcircuit. For the computer simulation, we fixed the parameters as $\beta = 3.0$ and $\gamma = 470.0$. Asymmetric attractors are distinguished with 2 colors in simulation result. In this study, we distinguish the attractors as follows:

(1) A Poincaré section is defined at $z_n = 1.0$ and $\dot{y}_n < 0$.
(2) When the solution hits the Poincaré section, and $y_n > 0.675$, the color is set as red and such state is defined state $\{M\}$. In the case of $y_n \leq 0.675$, the color is set as blue and defined state $\{P\}$.

The switching phenomenon is behavior that of attractor transition. This definition is important to grasp the domain of attractor in each subcircuit and as such, is being applied to following simulation results in this paper.

III. 2-TYPE CHAOS SYNCHRONIZATION

2-type synchronization phenomena depending on initial values can be observed in the system as shown in Fig. 1. Figure 3 shows the examples of synchronization phenomena. *N*-phase chaos synchronization is shown in Fig. 3 (a), and in-synchronization in Fig. 3 (b). Red and blue colors show the state $\{M\}$ and $\{P\}$ respectively. Both synchronization states are not synchronized rigorously since the system generates chaotic signal. However, those behaviour shown in Fig. 3 (a) and (b) is almost synchronized. Therefore these phenomena are called *N*-phase synchronization and synchronization respectively in this paper. It should be noted that when the subcircuits (n is odd number) take state $\{P\}$, while the others (n is even number) take state $\{M\}$ in Fig. 3. Namely, the subcircuits take opposite attractor state with respect to the neighbor subcircuits in this case. Figure 4 is simulation results in state of *N*-phase chaos synchronization. The phase difference between z_1 and z_n is enlarged from z_1 vs z_2 to z_1 vs z_6. Moreover, z_6, which are located diagonal position of z_1, is almost synchronized with anti-phase. This behaviour also can be observed in circuit experiment as shown in Fig. 5. Figure 6 shows time waveforms and Lissajous figure of z_1 and z_6 observed in simulation and circuit experiment. While z_1 takes state $\{P\}$, z_6 takes state $\{M\}$ and almost synchronized with anti-phase as shown in Fig. 6.

From this result, we can confirm that subcircuits which are located in diagonal position are synchronized with anti-phase in state of N-phase synchronization. Namely, sum of these time waveforms become almost 0, in case of that the number of the circuit is even number. Therefore, for observation of the propagation phenomena, we treat the case of the number of the circuit is even number in this paper.

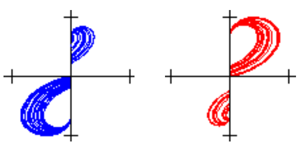


Fig. 2. Coexisting chaotic attractor observed in a subcircuit. Horizontal: y_1. Vertical: z_1. $\alpha = 0.35$, $\beta = 3.0$ and $\gamma = 470.0$.

IV. PROPAGATION OF SWITCHING PHENOMENA

Propagation pattern in case of $N = 6$ are shown in Fig. 7 (b). The attractors of the circuit located on diagonal position take state P, while the other take state M. These attractor states are propagated with clock rotation. The direction is determined by initial conditions. Figure 8 shows propagation velocity, which is expressed by $v = 1/T$. T is period of propagation derived by computer calculation. By increasing the coupling strength, the propagation velocity is also gradually increased and finally extinguished. Similarly, the propagation damping are induced by decreasing the coupling strength. One of the stable state of this system is shown in Fig. 7 (a). The subcircuits take opposite attractor with respect to neighboring subcircuits. These attractor states set shown in Fig. 7 (b) is stable N-phase synchronization state and propagation phenomena cannot be observed in any number of coupled chaotic circuit. Figure 9 shows propagation patterns in case of $N = 8$. Various stable propagation states are coexisting. Basically, the propagation velocity is almost constant for any propagation pattern. Figure 9 (b) is stable state which corresponds to the state of Fig 7 (a). Figure 9 (e) seems the propagation is extinguished, however, the system produces propagation with exact N-phase synchronization. All propagation patterns for $N = 6$ and $N = 8$ are shown in Table I. These patterns are depending on initial conditions. Moreover, in the case of uncoupled subcircuit, the circuit generates periodic orbit for the fixed parameter sets of $\alpha = 0.285$, $\beta = 3.0$ and $\gamma = 470$. It implies that the system in state of producing propagation is the states of torus or periodic orbit. With the increase in the number of the coupled chaotic circuits, propagation pattern increases exponentially. These propagation phenomena can be observed in large-scale case. Figure 10 shows an example of propagation phenomena in case of $N = 50$. The propagation phenomena can be observed clearly in case of large-scale case.

V. CONCLUSION

In this study, we have investigated propagation of switching phenomena on a ring of coupled chaotic circuits. As a result,

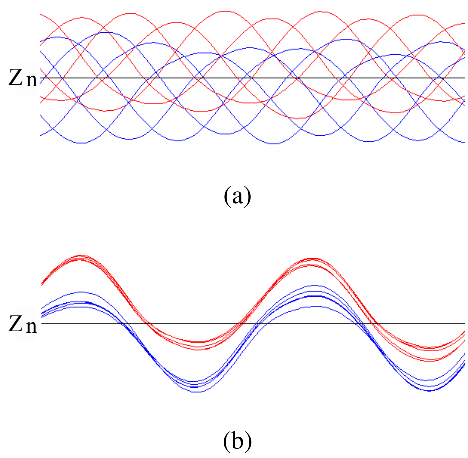


Fig. 3. 2-type synchronization phenomena depending on initial values. (a) *N*-phase chaos synchronization. (b) Synchronization. Vertical: z_n. Horizontal: τ. $N = 10$, $\alpha = 0.425$, $\beta = 3.0$, $\gamma = 470.0$ and $\delta = 0.20$.

TABLE I. STATE TABLE

State: S_{Nn}	CC_1	CC_2	CC_3	CC_4	CC_5	CC_6	Propagation
S_{61}	P or M	P or M	P or M	P or M	P or M	P or M	×
S_{62}	P(M)	M(P)	P(M)	M(P)	P(M)	M(P)	×
S_{63}	P(M)	M(P)	M(P)	P(M)	M(P)	M(P)	○

State	CC_1	CC_2	CC_3	CC_4	CC_5	CC_6	CC_7	CC_8	Propagation
S_{81}	P or M	P or M	P or M	P or M	P or M	P or M	P or M	P or M	×
S_{82}	P(M)	M(P)	P(M)	M(P)	P(M)	M(P)	P(M)	M(P)	×
S_{83}	P(M)	M(P)	M(P)	M(P)	P(M)	M(P)	M(P)	M(P)	○
S_{84}	P(M)	M(P)	P(M)	M(P)	P(M)	M(P)	M(P)	M(P)	○
S_{85}	P(M)	M(P)	M(P)	P(M)	M(P)	M(P)	M(P)	M(P)	○
S_{86}	P(M)	M(P)	P(M)	M(P)	M(P)	M(P)	M(P)	M(P)	○

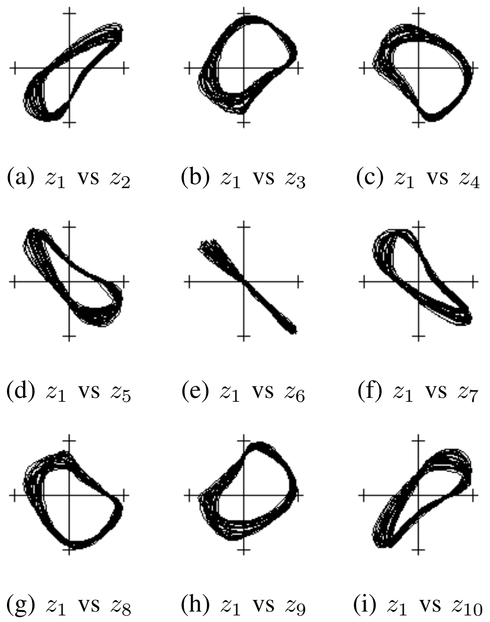


Fig. 4. Lissajous figures in state of N-phase chaos synchronization (computer simulations). z_1 vs z_n. $\alpha = 0.425$, $\beta = 3.0$, $\gamma = 470.0$ and $\delta = 0.2$.

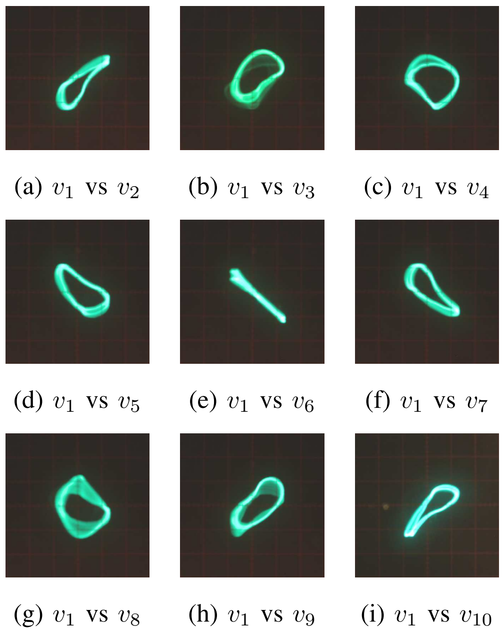


Fig. 5. Lissajous figures in state of N-phase chaos synchronization (circuit experiments). v_1 vs v_n[10V/div]. $L_1 = 625$[mH] $\pm 1\%$, $L_2 = 209$[mH] $\pm 1\%$, $C = 1.49$[nF] $\pm 4\%$ and $R = 100.0$[kΩ] $\pm 1\%$.

various patterns of propagation and relationships between propagation velocity and parameters have clarified. Moreover, with the increase in the number of coupled chaotic circuits, propagation pattern also increases exponentially. This result is extremely interesting and would give the effective suggestion to make clear the nonlinear phenomena in large-scale high-dimensional system. In our future work, we will investigate the mechanism of propagation of switching phenomena and regularity of the propagation patterns in proposed system.

REFERENCES

[1] L.O. Chua, M. Komuro and T. Matsumoto, “The doulbe scroll family”, IEEE Transactions on Circuit and Systems, vol. 33, no. 11, pp. 1073-1118, Oct. 1986.

[2] H. Sekiya, S. Mori and I. Sasase, “Synchronization of Self-Switching phenomena on Full-Coupled Chaotic Oscillators”, IEICE Trans., vol.J83-A, no. 11 pp. 1264-1275, 2000.

[3] T. Nishimoto, Y. Hosokawa and Y. Nishio, “Anti-Phase Synchronization of Switching Phenomena in Globally Coupled System of Chaotic Circuits”, IEICE Technical Report, no. NLP-48 & CAS2010-32, pp. 1-4, 2010.

[4] T. NISHIMOTO, Y. UWATE, Y. HOSOKAWA, Y. NISHIO and Daniele FOURNIER-PRUNARET ”N-Phase Synchronization of Asymmetric Attractors in a Ring of Coupled Chaotic Circuits” Proceedings of IEEE International Symposium on Circuits and Systems (ISCAS’14), pp. 1780-1783, Jun. 2014.

[5] Saori FUJIOKA, Yang YANG, Yoko UWATE and Yoshifumi NISHIO ”Double-Mode Simultaneous Oscillation in Three Coupled Hard Oscillators” Proceedings of RISP International Workshop on Nonlinear Circuits, Communications and Signal Processing (NCSP’12), pp. 72-75, Mar. 2012.

[6] Saori FUJIOKA, Yang YANG, Yoko UWATE and Yoshifumi NISHIO ”Asynchronous Oscillations of Double-Mode and N-Phase in a Ring of Simultaneous Oscillators” Proceedings of International Symposium on Nonlinear Theory and its Applications (NOLTA’13), pp. 459-462, Sep. 2013.

[7] Y. Nishio, K. Suzuki, S. Mori and A. Ushida, “Synchronization in Mutually Coupled Chaotic Circuits”, Proceedings of European Conference on Circuit Theory and Design(ECCTD’93), vol. 1, pp. 637-642, Aug. 1993.

[8] Y. Nishio, N. Inaba, S. Mori and T. Saito, “Rigorous Analyses of Windows in a Symmetric Circuit”, IEEE Transactions on Circuit and Systems, vol. 37, no. 4, pp. 473-487, Apr. 1990.

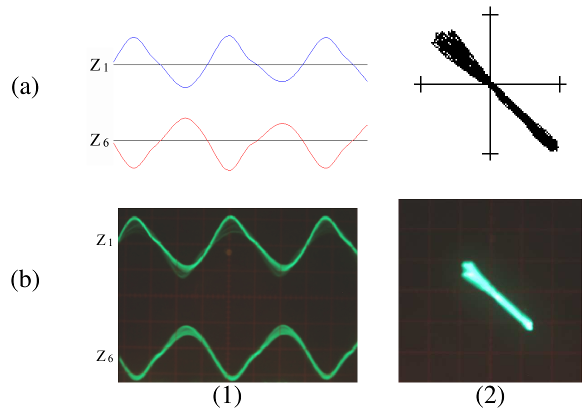
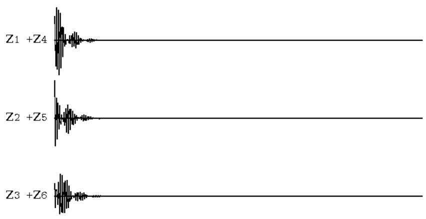
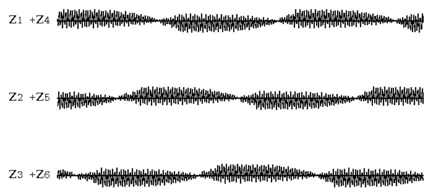


Fig. 6. Almost anti-phase synchronization state for $N = 10$. (a) Simulation result. (1) Time waveforms of z_1 and z_6. (2) z_1 vs z_6. (parameter values are the same as those in Fig. 4). (b) Circuit experiment. (1) Time waveforms of v_1 and v_6. (2) v_1 vs v_6. (parameter values are the same as those in Fig. 5).



(a) State:S_{62}.



(b) State:S_{63}.

Fig. 7. Propagation of switching phenomena for $N = 6$. $\alpha = 0.285$, $\beta = 3.0$ and $\delta = 0.068$.

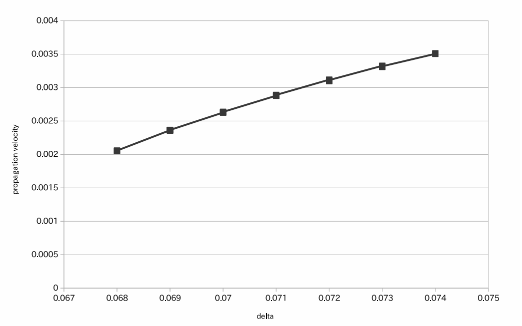
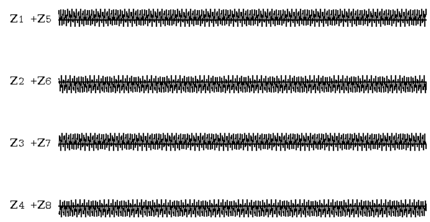
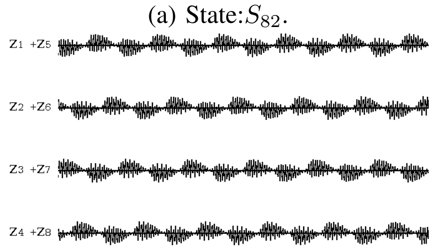


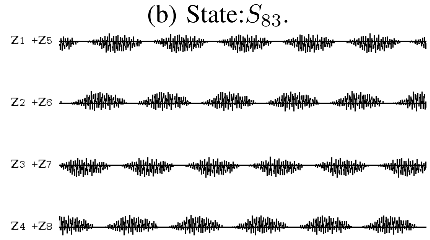
Fig. 8. Propagation velocity for $N = 6$. Vertical axis: v. Horizontal axis: δ.



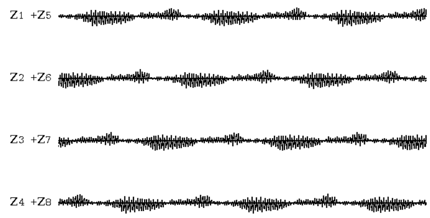
(a) State:S_{82}.



(b) State:S_{83}.



(c) State:S_{84}.



(d) State:S_{85}.

(e) State:S_{86}.

Fig. 9. Propagation of switching phenomena for $N = 8$. $\alpha = 0.285$, $\beta = 3.0$ and $\delta = 0.1$.

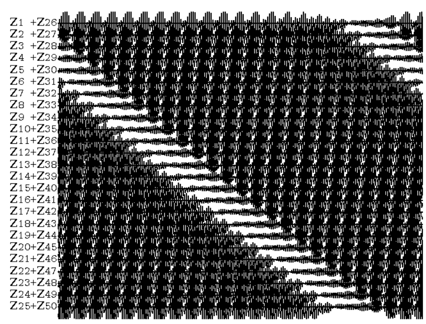


Fig. 10. An example of propagation of switching phenomena for $N = 50$. $\alpha = 0.285$, $\beta = 3.0$ and $\delta = 0.4$.