On Five Coupled van der Pol Oscillators with Different Oscillation Amplitudes

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***Abstract*—This article gives an explanation of the synchronization phenomena in a circuit which contains five van der Pol oscillators when some of them have different amplitudes from the others.**

I. Introduction

We are living in the world where there are so many example of synchronization: firefly luminescence, cry of birds and frogs, human applause, etc. Studies of synchronization phenomena have been reported in so many research of engineering filed: about synchronization in three coupled van der Pol oscillators with different coupling strength [1], about frustrated synchronization in coupled oscillator chains with unbalanced parametric distribution [2], synchronization-inspired partitioning and hierarchical clustering [3], about detection of synchronization phenomena in networks of hindmarsh-rose neuronal models [4], and also about synchronization in agents harvesting game with regional nature [5], etc. Furthermore, the applications of synchronization phenomena have also found in chemical, physical and biological fields: about chaos synchronization of chemical models [6], about synchronization and coupling analysis of applied cardiovascular physics in sleep medicine [7], or about multistate and multistage synchronization with excitatory chemical and electrical synapses [8], etc.

In the past research, we have proposed a new type of coupled van der Pol oscillators system and discovered that in 3 coupled oscillators system, if we changed the amplitude of one oscillator to some appropriate values, we can observe the synchronization phenomena by computer simulation, and proved these results by both averaging method and circuit experimental results [9]. For 4 coupled oscillators system, we have changed the amplitude of not only 1 but also 2 and 3 oscillators and introduced the computer simulation results of synchronization phenomena in [10].

In this article, in order to expand the number of oscillators, we give some computer simulation results about synchronization phenomena of 5 coupled van der Pol oscillators system and compare them to obtained results in [9] and [10].

II. Circuit Model [9][10]

In our study, $n > 2$ van der Pol oscillators are used to build the circuit model (Fig. 1). One van der Pol oscillator contains one capacitor C, one inductor L, and one non-linear resistor.

The circuit equations are described as follows:

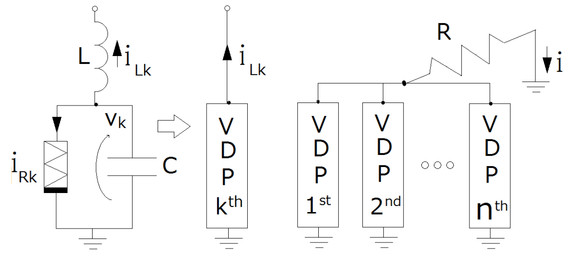


Fig. 1. Circuit model.

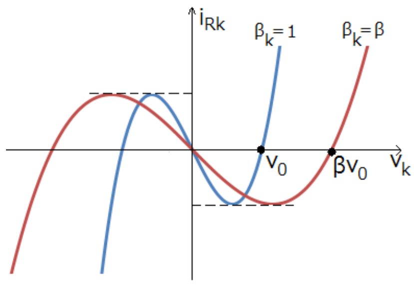


Fig. 2. Relation between amplitude and β_k.

$$C\frac{dv_k}{dt} = -i_{Lk} - i_{Rk} \quad (1)$$

$$L\frac{di_{Lk}}{dt} = -v_k - R\sum_{j=1}^{n} i_{Lj} \quad (2)$$

The $v - i$ characteristics of the nonlinear resistor are expressed by:

$$i_{Rk} = -g_1\left(\frac{v_k}{\beta_k}\right) + g_3\left(\frac{v_k}{\beta_k}\right)^3, \quad k = 1, .., n. \quad (3)$$

Figure 2 shows the relation between amplitude of the k-th oscillator and β_k. Generally, $\beta_k = 1$ mean that the amplitude of k-th oscillator is kept in the normal situation, while in the other cases, the amplitude of k-th oscillator is changed.

To normalize circuit equations, we use Eqs. (4)-(6) to change the variables:

$$t = \sqrt{LC}\tau \quad (4)$$

$$v_k = \sqrt{\frac{g_1}{3g_3}}x_k, \; i_{Lk} = \sqrt{\frac{Cg_1}{3Lg_3}}y_k \quad (5)$$

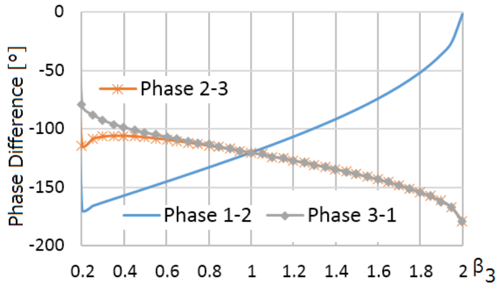


Fig. 3. Phase difference result when $n = 3$.

$$\alpha = R\sqrt{\frac{C}{L}},\ \varepsilon = g_1\sqrt{\frac{L}{C}} \tag{6}$$

With the new variables, Eqs. (1)-(3) are normalized as:

$$\frac{dx_k}{d\tau} = \frac{\varepsilon}{\beta_k}x_k\left(1-\frac{1}{3\beta_k^2}x^2{}_k\right)-y_k \tag{7}$$

$$\frac{dy_k}{d\tau} = x_k-\alpha\sum_{j=1}^{n}y_j \quad (k=1,..,n) \tag{8}$$

where α is the coupling factor and ε is the strength of nonlinearity.

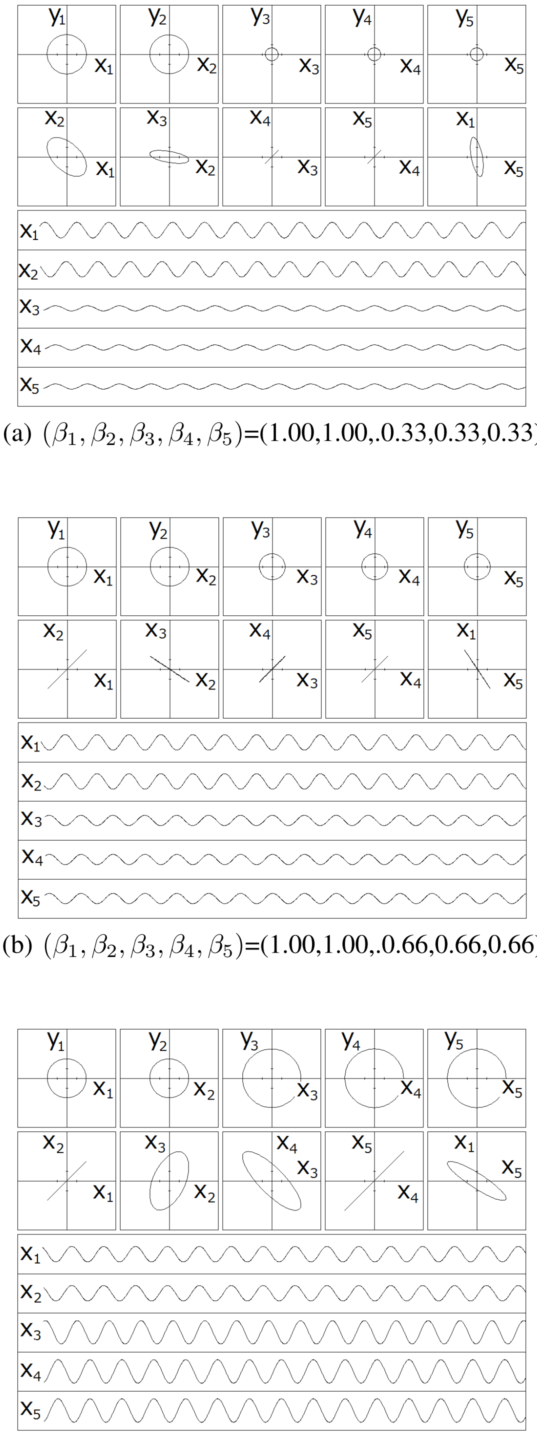
III. Synchronization for n=3 [9]

As the results of our past article [9], in 3 coupled oscillators system, if we change only 3rd oscillator's amplitude (in other words, we change only β_3 while keeping $\beta_1 = \beta_2 = 1$), we can obtain the result about phase difference as shown in Fig. 3. This result was obtained by both computer simulation method, theoretical analysis method and was proved by some circuit experimentals. We will use this to compare with 5 coupled oscillators system.

IV. Synchronization for $n = 5$

In 5 oscillators system, we first divide them to 2 groups of amplitude: $\beta_1 = \beta_2 = 1$ and $\beta_3 = \beta_4 = \beta_5$ are changed and obtain computer simulation results as shown in Fig. 4. In Fig. 4(a), when the oscillator 3rd, 4th, and 5th have same phase and their sum can be concerned as one oscillator with 3 times bigger of amplitude. By this way, we can concerned the system as 3 oscillators system which have 3 same amplitudes, and these obtained results are similar to Fig. 3 when $\beta_3 = 1$. Similarly, the phase differences shown in Fig. 4(b) are same with the phase difference shown in Fig. 3 when $\beta_3 = 2$: 1st and 2nd oscillator are in-phase synchronization, and each one of them is anti-phase synchronization with 3rd, 4th, or 5th oscillator.

When we divide the system to 3 groups of amplitude: $\beta_1 = 1, \beta_2 = \beta_3$ and $\beta_4 = \beta_5$, and change both of β_3 and β_5 separately, we obtain the computer simulation results as shown in Figs. 5. The phase relation in Fig. 5(a), Fig. 5(b) can be compared with the phase relation in Fig. 3 when β_3 equals 1, 2, respectively. For more details, Fig. 6 shows when synchronization phenomena occurs. In Fig. 6, the step of changing β_3, β_5 is 0.25. Currently, mathematical difficulty

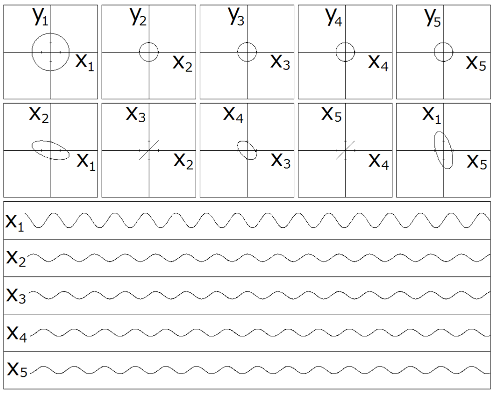


(a) $(\beta_1,\beta_2,\beta_3,\beta_4,\beta_5)$=(1.00,1.00,.0.33,0.33,0.33)

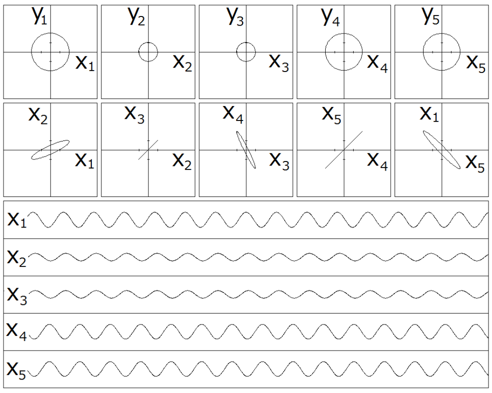
(b) $(\beta_1,\beta_2,\beta_3,\beta_4,\beta_5)$=(1.00,1.00,.0.66,0.66,0.66)

(c) $(\beta_1,\beta_2,\beta_3,\beta_4,\beta_5)$=(1.00,1.00,.1.50,1.50,1.50)

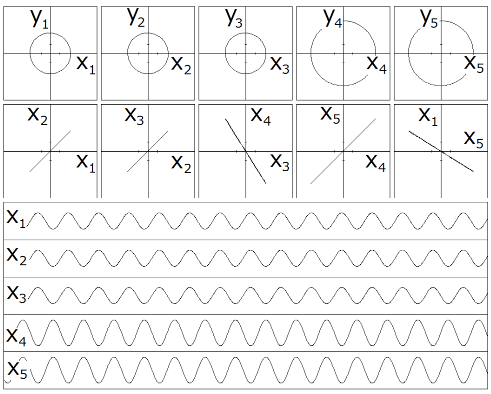
Fig. 4. Simulation results when dividing oscillators into 2 group.



(a) $(\beta_1, \beta_2, \beta_3, \beta_4, \beta_5)$=(1.0,0.5,.0.5,0.5,0.5)



(b) $(\beta_1, \beta_2, \beta_3, \beta_4, \beta_5)$=(1.0,0.5,0.5,1.0,1.0)



(c) $(\beta_1, \beta_2, \beta_3, \beta_4, \beta_5)$=(1.0,1.0,1.0,1.5,1.5)

Fig. 5. Simulation results when dividing oscillators into 3 group.

does not allow us to prove theoretically our results. However, the obtained results in this article have some similarities with the proved results in [9], so we can consider that they are credible.

As an idea, in larger coupled system, by dividing them to 2 or 3 groups we can estimate phase difference between each phase of oscillators.

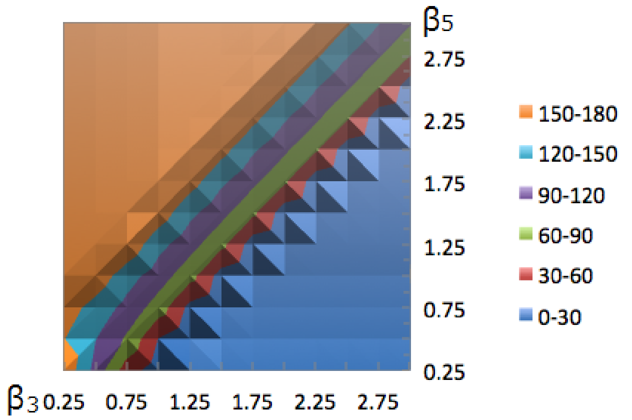


Fig. 6. Distribution map of synchronization between phase 1-2 when $n = 5$.

V. Conclusions

As development and extending of our past results reported in [9] and [10], in this article we have concerned about the synchronization phenomena in 5 coupled van der Pol oscillators system, when a part of them have different amplitudes. In the next step, it is necessary to complete theoretical analysis and circuit experiment to perfectly verify obtained results in 5 coupled oscillators system.

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