Investigation of Partial Synchronization in Coupled Chaotic Circuit Network with Local Bridge

Kenta Ago, Yoko Uwate and Yoshifumi Nishio
Dept. of Electrical and Electronic Engineering, Tokushima University
2-1 Minami-Josanjima, Tokushima 770–8506, Japan
Email: {kentago,uwate,nishio}@ee.tokushima-u.ac.jp

***Abstract*—In our previous work, the influence of local bridge on a complex network of 25 coupled chaotic circuits has been investigated. In this study, we focus on partial synchronization in the case of 5 clusters that are divided from local bridges. By means of computer simulations, the states of significant node is located in between two local bridges in the network and partial synchronization probability of each cluster are investigated. Moreover, we statistically analyze the sojourn time of synchronization in various sizes of the clusters.**

I. Introduction

Complex networks have attracted a great deal of attention from various fields since the discovery of "small-world" network [1] and "scale-free" network [2]. In particular, how network topological structure influences its dynamical behaviors, is a hot topic for understanding the structural function on the networks and suitable for practical application in many disciplines such as biology, sociology and economics. Most of complex networks can be represented as a graph by nodes (vertices) and edges (links). On the other hand, synchronization is one of the typical phenomena in the field of natural science. In particular, synchronization phenomena on the networks of coupled chaotic systems are very interesting. However, there are not many studies for complex networks of continuous-time real physical systems such as electrical circuits.

In sociology, there is a famous theory called "The strength of weak ties" by Granovetter [3]. "Local bridge" is the edge which predominantly shorter than other routes between two nodes, and is essential for information propagation in complex networks. In our previous work, we have investigated the influence of local bridge on a complex network by means of synchronization phenomena of 25 coupled chaotic circuits [4]. Local bridges almost behave asynchronously, partial synchronization in the network is almost occurred from local bridge.

In this study, partial synchronization in a complex network with local bridge of 25 coupled chaotic circuits is investigated. We focus on partial synchronization in the case of 5 clusters that are divided from local bridges. For the investigations, we define the synchronization and a time interval. By means of computer simulations, the states of significant node is located in between two local bridges in the network and partial synchronization probability of each cluster are investigated. When partial synchronization probabilities of each cluster are evaluated, we consider the typical three structural metrics of each cluster. Moreover, we statistically analyze the sojourn time of synchronization in various sizes of the clusters.

II. Network model

Figure 1 shows the chaotic circuit which is three-dimensional autonomous circuit proposed by Shinriki *et al.* [5][6]. This circuit is composed by an inductor, a negative resistor, two capacitors and dual-directional diodes. A proposed network model of 25 coupled chaotic circuits with local bridge is shown in Fig. 2. In this model, chaotic circuits are applied to each node of the network and each edge corresponds to a coupling resistor R. In this model, local bridges are considered to be the edges 1–25, 8–9, 14–15, 15–16 and 22–23.

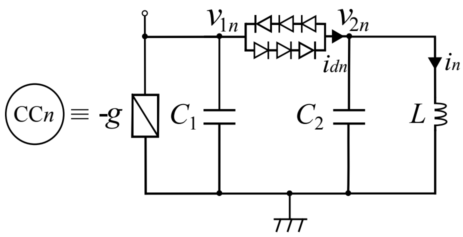

Fig. 1. Chaotic circuit.

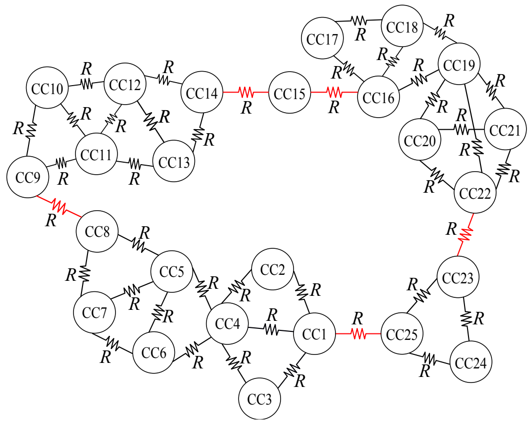

Fig. 2. Network model.

First, the circuit equations are given as follows:

$$\begin{cases} L\dfrac{di_n}{dt} &= v_{2n} \\ C_1\dfrac{dv_{1n}}{dt} &= gv_{1n} - i_{dn} - \dfrac{1}{R}\displaystyle\sum_{k\in S_n}(v_{1n} - v_{1k}) \\ C_2\dfrac{dv_{2n}}{dt} &= -i_n + i_{dn}, \end{cases} \qquad (1)$$

where $n = 1, 2, 3, ..., 25$ and S_n is the set of nodes which are directly connected to the node n. We approximate the $i - v$ characteristics of the nonlinear resistor consisting of the diodes by the following three-segment piecewise-linear function:

$$i_{dn} = \begin{cases} G_d(v_{1n} - v_{2n} - V) & (v_{1n} - v_{2n} > V) \\ 0 & (|v_{1n} - v_{2n}| \le V) \\ G_d(v_{1n} - v_{2n} + V) & (v_{1n} - v_{2n} < -V). \end{cases} \qquad (2)$$

By using the parameters and the variables:

$$\begin{cases} i_n = \sqrt{\dfrac{C_2}{L}}Vx_n,\ v_{1n} = Vy_n,\ v_{2n} = Vz_n \\ t = \sqrt{LC_2}\tau,\ \text{“}\cdot\text{”} = \dfrac{d}{d\tau},\ \alpha = \dfrac{C_2}{C_1} \\ \beta = \sqrt{\dfrac{L}{C_2}}G_d,\ \gamma = \sqrt{\dfrac{L}{C_2}}g,\ \delta = \dfrac{1}{R}\sqrt{\dfrac{L}{C_2}}, \end{cases} \qquad (3)$$

the normalized circuit equations are given as follows:

$$\begin{cases} \dot{x_n} &= z_n \\ \dot{y_n} &= \alpha\gamma y_n - \alpha f(y_n - z_n) - \alpha\delta\displaystyle\sum_{k\in S_n}(y_n - y_k) \\ \dot{z_n} &= f(y_n - z_n) - x_n. \end{cases} \qquad (4)$$

The nonlinear function $f()$ corresponds to the $i - v$ characteristics of the nonlinear resistances consisting of the diodes and are described as follows:

$$f(y_n - z_n) = \begin{cases} \beta(y_n - z_n - 1) & (y_n - z_n > 1) \\ 0 & (|y_n - z_n| \le 1) \\ \beta(y_n - z_n + 1) & (y_n - z_n < -1). \end{cases} \qquad (5)$$

This circuit generates asymmetric chaotic attractor as shown in Fig. 3. The values y and z in Fig. 3 correspond to v_1 and v_2 of the circuit in Fig. 1, respectively.

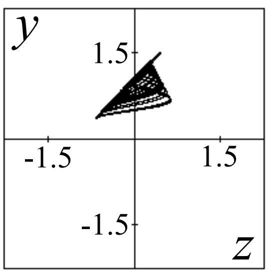

Fig. 3. Example of chaotic attractor ($\alpha = 0.4$, $\beta = 20$ and $\gamma = 0.5$).

III. Synchronizations states

In this research, we fix the circuit parameters as $\alpha = 0.4$, $\beta = 20$, $\gamma = 0.5$ and $\delta = 1.0$ for all chaotic circuits. Each circuit is given different initial values for computer simulations. Figure 4 shows an example of computer simulation results. The vertical axes denote the differences between the voltages (corresponding to v_1 of the circuit in Fig. 1) of the two chaotic circuits. Namely, if the two chaotic circuits are synchronized, the value of the graph should be almost zero like 19-20. We can confirm that synchronizations of local bridges (8-9, 14-15, 15-16, 22-23 and 1-25) are easy to break down compared with others. In this paper, we define two synchronization states of “global synchronization” and “partial synchronization”. Figure 4 shows that the network switches to two synchronization states. Additionally, partial synchronization is almost occurred from local bridges.

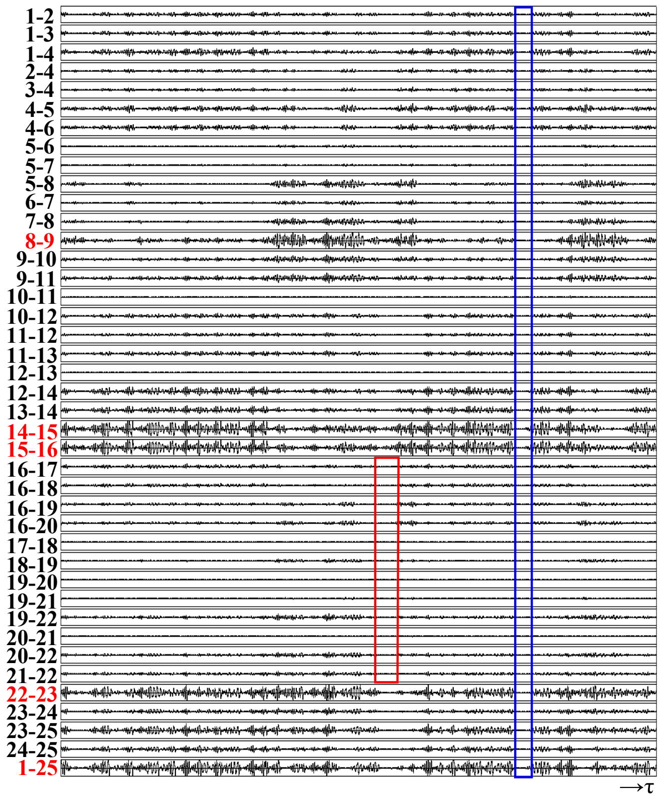

Fig. 4. Phase difference waveform ($\delta = 1.0$).

In order to analyze synchronization states, we define the synchronization by the following equation:

$$|y_n - y_k| < 0.01 \quad (k \in S_n). \qquad (6)$$

We fix a certain time interval as ($\tau = 10,000$ and $step = 0.01\tau$) and statistically investigate partial synchronization in each cluster of 25 coupled chaotic circuits by means of the above definition of the synchronization.

IV. Partial Synchronization

In this section, we consider partial synchronization in the case of 5 clusters (nodes 1∼8, 9∼14, 15, 16∼22 and 23∼25) that are divided from local bridges. Then, the network is shown in Fig. 5.

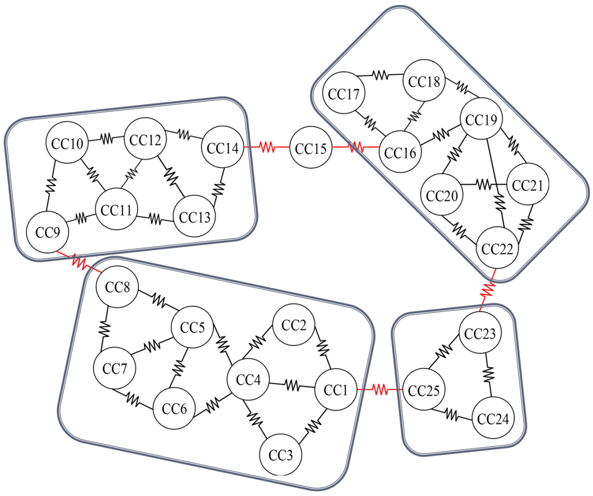

Fig. 5. A partial synchronization state from local bridges.

First, node 15 is located in between two local bridges (14-15 and 15-16) in the network. Therefore the investigating of the states of significant node 15 in the network, is important. Figure 6 shows the distribution of the states of node 15 during the time interval. In this figure, the state A shows that node 15 are not synchronized both nodes 14 and 15. The state B shows that nodes 14, 15 and 16 are all synchronized. The state C shows that nodes 15 and 16 are synchronized however nodes 14 and 15 are not synchronized. The state D shows the opposite the state C. In four states, state A is the highest distribution. Thus, we consider that node 15 faces with dilemma between two clusters, and hence tend to be isolated. However, state B is the second highest distribution that is interested.

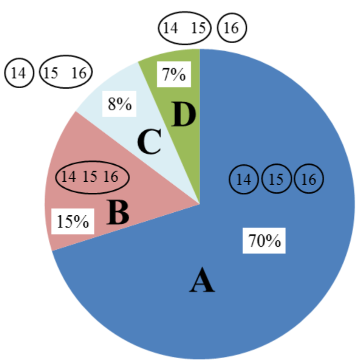

Fig. 6. Distribution of the states of node 15.

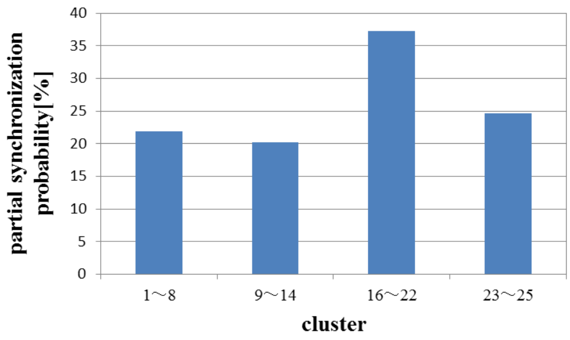

Fig. 7. Partial synchronization probability of each cluster.

Next, we statistically investigate partial synchronization in each cluster of the network by means of the above definition of the synchronization. Figure 7 shows the partial synchronization probability of each cluster during the time interval. We consider that the partial synchronization probabilities of these clusters depend on network topological structures. Topological structures in complex networks can be evaluated by the typical three structural metrics (degree, clustering coefficient and path length). Degree shows the number of edges on a node. Clustering coefficient shows the number of actual links between neighbors of a node divided by the number of possible links between those neighbors. Path length shows the shortest path in the network between two nodes. Table I shows the properties of each cluster. In Table I, N denotes the number of nodes, E denotes the number of edges, k denotes the average degree, C denotes the average of clustering coefficient and L denotes the average of path length. We assume that the result of Fig. 7 depends on three structural metrics in Table I. In particular, partial synchronization probability of cluster 16∼22 is most highest in 4 clusters. The structural metrics k and C of cluster 16∼22 show the largest values in 4 clusters. Also, the path length of cluster 16∼22 is the second shortest in 4 clusters. Thereby, we consider that the community strength characterized by large clustering coefficient and short path length can be evaluated by partial synchronization probability of each cluster.

TABLE I
Properties of each cluster.

Cluster	N	E	k	C	L
1∼8	8	12	3.25	0.60	1.79
9∼14	6	9	3.17	0.50	1.57
16∼22	7	12	3.43	0.66	1.52
23∼25	3	3	2.67	0.56	1.00

Moreover, we focus on the sojourn time of partial synchronization in each cluster. Figure 8 shows the distribution of the sojourn time of synchronization in various sizes of the clusters. The cluster sizes denote the number of node in each cluster. The slots in the horizontal axes of the figure denote the ranges of the sojourn time in Table II. Figure 8(a) shows the 1 cluster state that the entire network are synchronized. From Fig. 8(a), the ratio of slot 1 is predominantly high. Namely, synchronization of the entire network tend to disappear immediately. From Figs. 8(b), (c) and (e), these graphs are very similar and the ratio of slot 1 is accounted for a substantial fraction of distribution. However, the graph of cluster 16∼22 shows that the ratio of slot 3 is the highest in 6 slots from Fig. 8(d), hence the sojourn time of synchronization of cluster 16∼22 is longer than other clusters. Namely, cluster 16∼22 can be considered as a robust cluster.

TABLE II
RANGES OF SLOTS IN FIG. 8.

Slot	Sojourn time (τ)
1	τ<1
2	1 $\leq\tau$< 2
3	2 $\leq\tau$< 3
4	3 $\leq\tau$<4
5	4 $\leq\tau$<5
6	τ $\geq$5

V. CONCLUSION

In this study, a complex network of 25 coupled chaotic circuits with local bridge has been analyzed based on synchronization phenomena. The nodes and edges on the network correspond to the chaotic circuits and coupling resistors, respectively. By means of computer simulations, partial synchronization is almost occurred from local bridges. Therefore we investigated partial synchronization in the case of 5 clusters that are divided from local bridges. As the results, significant node is located in between two local bridges in the network that tend to be isolated and partial synchronization probability of each cluster almost depends on three structural metrics. From these results, we consider that local bridge is suitable for finding cluster and significant node on complex networks and partial synchronization probability of each cluster can be effective for evaluating as the community strength on complex networks. In order to understand the phenomena correctly, more detailed investigation considering more various clusters should be carried out in our future works.

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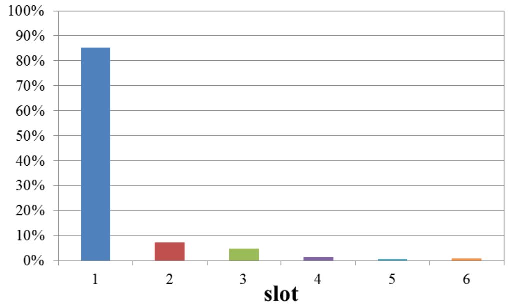
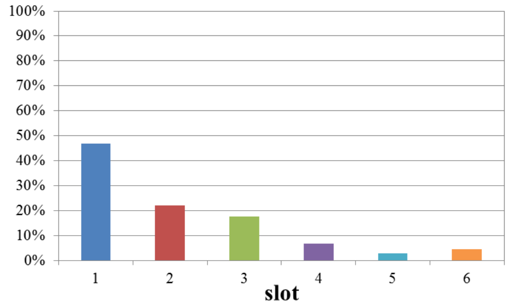
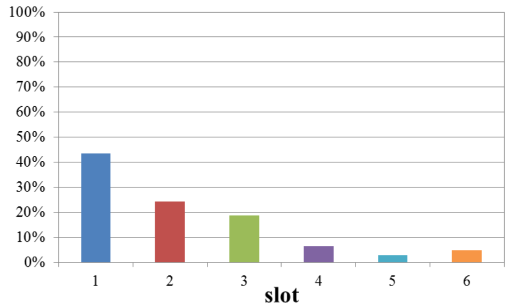


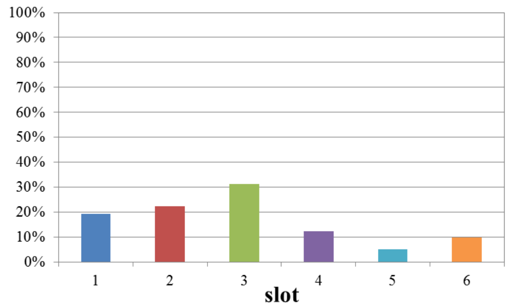
(a) 1∼25 (Cluster size = 25).



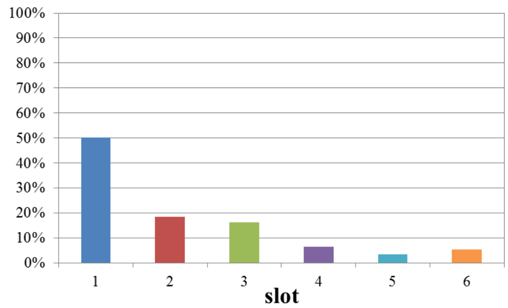
(b) 1∼8 (Cluster size = 8).



(c) 9∼14 (Cluster size = 6).



(d) 16∼22 (Cluster size = 7).



(e) 23∼25 (Cluster size = 3).

Fig. 8. Distribution of the sojourn time of synchronization in various sizes of the clusters.