Synchronization Pattern of Chaotic System Containing Time Delay

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***Abstract*—In this study, we investigate synchronization phenomena observed in coupled chaotic circuits containing time delay. Various synchronization states can be observed . Especially, we focus on relationships between synchronization state and parameters. Coexisting synchronization states depending on initial values can be observed in the proposed system by carrying out computer calculation. Moreover, we investigate the effect of chaotic behaviour on subcircuit to synchronization states. Chaotic strength of subcircuit possibly induces anti-phase synchronization.**

I. Introduction

Recently, researches on chaos phenomena have been attracting attention. The chaos phenomenon shows a complex unpredictable behavior. It can be observed in various science fields, such as biology, economics, physics and astronomy. A number of studies on synchronization of coupled chaotic circuits have been made [1]. The synchronization is one of the typical phenomena observed in nature. There are flashing of fireflies, rhythm of the heart cells and laser oscillation in examples of the synchronization. In addition, synchronization phenomenon, there is an advantage that a large energy is obtained by synchronizing the small energy. By utilizing this property, it is possible to realize a pacemaker that can control the synchronization of the cardiomyocytes of the heart, which is actually used as a medical device. On the other hand, electric circuit observed the chaotic phenomena. There are a lot of merits using electric circuit to investigate chaotic behaviour. For instance, inexpensive element, short experiment time, high repeatability of the experiment and so on. There are various types of chaotic circuits. We focus on the chaos circuit including a also time delay in that [2]-[4].

In this study, we investigate two coupled chaotic circuits containing time delay. Especially we focus on the synchronization phenomena of our proposed circuit. By carrying out computer simulations, two types of synchronization phenomena depending on initial values can be observed. Moreover, we investigate the relationships between parameters and synchronization phenomena.

II. Circuit Model

The chaotic circuit used in this study is shown in Fig. 1. The circuit is composed of a $-gLC$ oscillator and the amplitude control loop. $-g$ is a negative resistance, the current flowing through the inductor L is i, the voltage between the capacitor C is v. Switching operation is shown in Fig. 2, it controls the amplitude of the oscillator. The switching controls the amplitude of v by changing from the negative resistance to the positive resistance when the amplitude is larger than the threshold value. This switching operation is included time delay. T_d denotes the time delay. First, the switch is connected to a negative resistance. v is amplified while oscillating, the amplitude is bigger a certain threshold voltage V_{th}. The switch does not immediately connect in the positive resistance however the switch is connected after T_d seconds. In the computer simulation, T_d is set to $\pi/(1-\alpha^2)$.

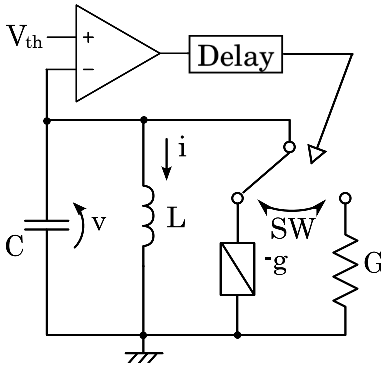

Fig. 1. Circuit model.

The circuit equations of the system are given as follow:

(A) In case of switch connected to -g,

$$\begin{cases} L\dfrac{di}{dt} &= v \\ C\dfrac{dv}{dt} &= gv - i. \end{cases} \tag{1}$$

(B) In case of switch connected to G,

$$\begin{cases} L\dfrac{di}{dt} &= v \\ C\dfrac{dv}{dt} &= -Gv - i. \end{cases} \tag{2}$$

By changing the parameters and variable as follows:

$$i = \sqrt{\frac{C}{L}}V_{th}x,\ v = V_{th}y,\ t = \sqrt{LC}\tau,$$

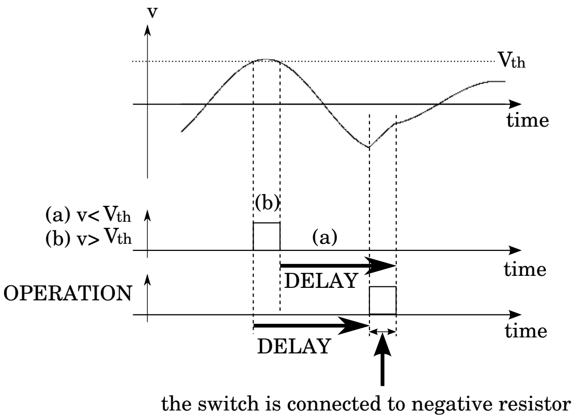

Fig. 2. Switching operation.

$$g\sqrt{\frac{C}{L}} = 2\alpha \text{ and } G\sqrt{\frac{C}{L}} = 2\beta.$$

The normalized circuit equations of the system are given as follow:

(A) In case of switch connected to $-g$,

$$\begin{cases} \dot{x} &= y \\ \dot{y} &= 2\alpha y - x. \end{cases} \tag{3}$$

(B) In case of switch connected to G,

$$\begin{cases} \dot{x} &= y \\ \dot{y} &= -2\beta y - x. \end{cases} \tag{4}$$

III. Two coupled chaotic circuits

Figure 3 shows the schematic of two chaotic circuits containing time delay coupled by a inductor. By changing the parameters and variables as follows:

$$i_n = \sqrt{\frac{C}{L}} V_{th} x_n, \; v_n = V_{th} y_n, \; t = \sqrt{LC}\tau,$$
$$g\sqrt{\frac{C}{L}} = 2\alpha, \; G\sqrt{\frac{C}{L}} = 2\beta \text{ and } \gamma = \frac{L}{L_0};$$

The normalized circuit equations of the system are given as follows:

(A) In case of the switch is connected to $-g$,

$$\begin{cases} \dot{x}_n &= y_n \\ \dot{y}_n &= 2\alpha y_n - x_n - \gamma(x_{n+1} - x_n). \end{cases} \tag{5}$$

(B) In case of switch connected to G,

$$\begin{cases} \dot{x}_n &= y_n \\ \dot{y}_n &= -2\beta y_n - x_n - \gamma(x_{n+1} - x_n). \end{cases} \tag{6}$$

where

$$(n = 1, 2), \quad x_3 = x_1.$$

Figure 4 shows simulation results in case of two coupled chaotic circuits. Figure 4 (1) and (2) are two types of synchronization phenomena depending on the initial values which are corresponding to almost in-phase synchronization and anti-phase synchronization states, respectively. By increasing the parameter β, anti-phase synchronization phenomena are induced and in-phase synchronization gradually become unstable. The parameter β qualitatively denotes the chaotic strength. Namely, it is considered that anti-phase synchronization state is induced by chaotic behavior of each subcircuit as shown in Fig. 4 (3).

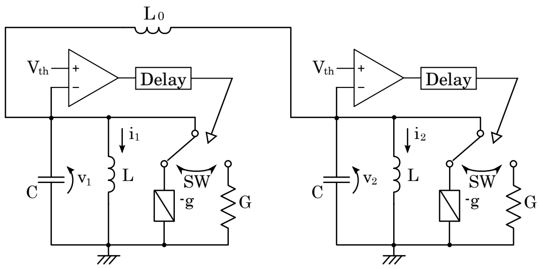

Fig. 3. A coupled 2 chaotic circuits containing time delay

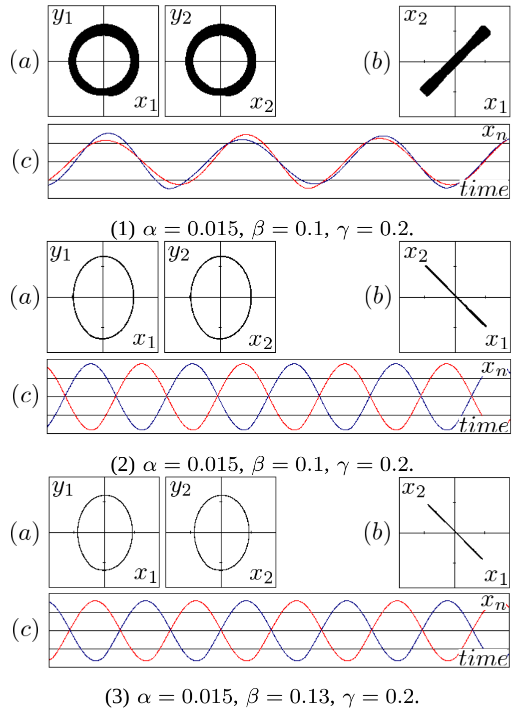

Fig. 4. Simulation results of N = 2. (a) Attractor. (b) Lissajous figure. (c) Timewaveform. Red and blue colors denote x_1 and x_2 respectively.

IV. Conclusion

In this study, we have investigated synchronization phenomena observed by coupled chaotic circuits containing time delay. As a result, induction of anti-phase synchronization caused by chaotic strength of subcircuit have clarified. In our future work, we will investigate the parameter region of coexisting synchronization and the mechanism of induction of anti-phase synchronization phenomena.

References

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