Synchronization Phenomena of Parametrically Excited Oscillators with Small Mismatch in Random Network

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***Abstract*—In this study, we investigate synchronization phenomena of parametrically excited van der Pol oscillators with small mismatch. By increasing the small mismatch, we observe unsynchronous phenomena. Furthermore, we apply this circuit model to ten coupled oscillators as random network model.**

I. INTRODUCTION

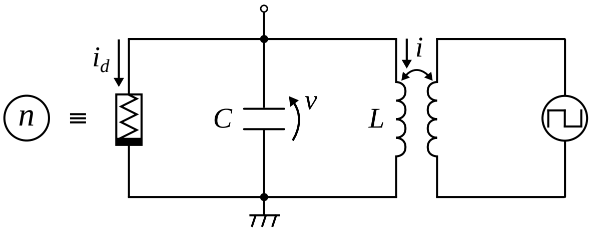
Synchronization phenomena is one of the fundamental phenomena in nature and it is observed over the various fields. Studies on synchronization phenomena of coupled oscillators are extensively carried out in various fields, physics [1], biology [2], engineering and so on. We consider that it is important to investigate the synchronization phenomena of coupled oscillators for the future engineering application. The coupled van der Pol oscillator is one of coupled oscillators, and synchronization generated in the system can model certain synchronization of natural rhythm phenomena. The van der Pol oscillator is studied well because it is expressed in simple circuit. Parametric excitation circuit is one of resonant circuits, and it is important to investigate various nonlinear phenomena of the parametric excitation circuits for future engineering applications. In simple oscillator including parametric excitation, Ref. [3] reports that the almost periodic oscillation occurs if nonlinear inductor has saturation characteristic. Additionally the occurrence of chaos is referenced in Refs. [4] and [5].

In our previous study, we have investigated synchronization of parametrically excited van der Pol oscillators [6]. By carrying out computer calculations for two or three subcircuits case, we have confirmed that various kinds of synchronization phenomena of chaos are observed. In the case of two subcircuits, the anti-phase synchronization is observed. In the case of three subcircuits, self-switching phenomenon of synchronization states is observed. Furthermore, we investigate synchronization of parametrically excited van der Pol oscillators with small mismatch. The small mismatch is added to the amplitude of the function relating to parametrically excitation. In the case of two subcircuits, we confirm that the two subcircuits are synchronized at in-phase state when the adding mismatch is small. By increasing the small mismatch, we observe unsynchronous phenomena. Additionally, we applied this circuit model to ten coupled oscillators as random network model with hub [7].

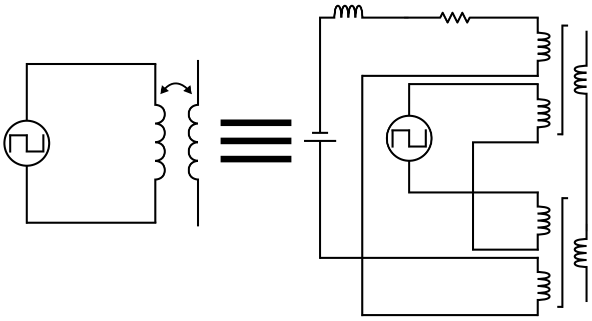
In this study, we investigate synchronization of this circuit model to ten coupled oscillators as random network model without hub.

II. VAN DER POL OSCILLATOR UNDER PARAMETRIC EXCITATION

The circuit model of van der Pol oscillator under parametric excitation is shown in Fig. 1.



(a) Parametrically excited van der Pol oscillator



(b) Time-varying inductor

Fig. 1. Circuit model.

The circuit includes a time-varying inductor L whose characteristics are given as the following equation. The time-varying inductor is shown as Fig. 2.

$$L = L_0\gamma(\tau). \tag{1}$$

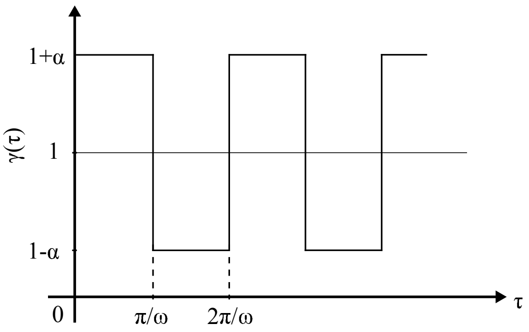


Fig. 2. Function relating to parametrically excitation.

$\gamma(\tau)$ is expressed in a rectangular wave as shown in Fig. 2, and its amplitude and angular frequency are termed α and ω, respectively. The $v-i$ characteristics of the nonlinear resistor are approximated by the following equation.

$$i_d = -g_1 v_k + g_3 v_k. \tag{2}$$

By changing the variables and the parameters,

$$\begin{cases} t = \sqrt{L_0 C}\tau, \;\; v_n = \sqrt{\dfrac{g_1}{g_3}} x_n \\ \omega = \omega_0 \sqrt{L_0 C} \\ i_n = \sqrt{\dfrac{g_1}{g_3}} \sqrt{\dfrac{C}{L_0}} y_n \\ \varepsilon = g_1 \sqrt{\dfrac{L_0}{C}}, \;\; \delta = \dfrac{1}{R}\sqrt{\dfrac{L}{C}} \end{cases} \tag{3}$$

The normalized circuit equations are given by the following equations.

$$\begin{cases} \dfrac{dx_n}{d\tau} = \varepsilon(x_n - x_n^3) - y_n + \delta \displaystyle\sum_{k \in S_n} (x_k - x_n) \\ \dfrac{dy_n}{d\tau} = \dfrac{1}{\gamma(\tau)} x_n \end{cases} \tag{4}$$

where $n = 1, 2, 3, ..., 10$. S_n is the set of nodes which are directly connected to the node n.

III. Random Network Case

We investigate synchronization phenomena of ten coupled oscillators as random network model without hub. Ten coupled oscillator model is shown in Fig. 3. In this figure, the model does not have hub. Here, the small mismatch is added to α which is corresponding to the amplitude of the function relating to parametrically excitation. The mismatch is generated by random and the range of the mismatch is set to [-0.01:0.01]. Table I shows the pattern of five types of the small mismatch used in this computer simulations.

Figure 4 shows the simulation results. The horizontal axis denotes the coupling strength δ, and the vertical axis denotes α of the second oscillator. In this graph, the lower area of each line denotes synchronous area and the upper area of each line denotes unsynchronous area. By increasing the value of the coupling strength, the synchronous area becomes large. We can see that the synchronous area is determined by the small mismatch pattern. Namely, several types of synchronization states can be observed from ten coupled oscillators with the small mismatch.

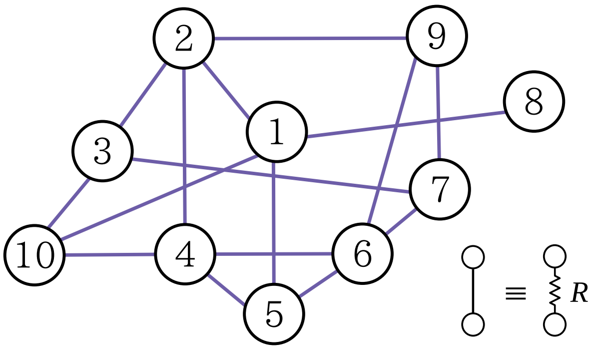


Fig. 3. A random network model without hub.

TABLE I
SMALL MISMATCH

small mismatch	w=1	w=2	w=3	w=4	w=5
α_1	0	-0.001	0.009	0.003	-0.006
α_2	-0.009	0.008	0.008	-0.01	-0.009
α_3	-0.006	-0.006	0.009	0.009	0.01
α_4	-0.001	-0.005	-0.01	-0.003	-0.007
α_5	0.009	0.007	-0.01	0.001	-0.001
α_6	-0.002	-0.01	-0.004	-0.002	0.004
α_7	0	-0.001	-0.003	0.007	0.009
α_8	0	-0.004	0.005	-0.006	0.007
α_9	-0.001	-0.01	0.009	-0.01	0.005
α_{10}	0.005	0.009	-0.009	-0.009	-0.001

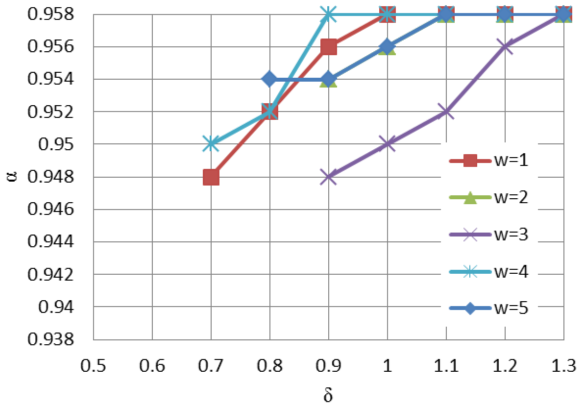


Fig. 4. Synchronous state with δ.

IV. CONCLUSIONS

In this study, we have investigated synchronization phenomena of parametrically excited van der Pol oscillators with small mismatch. In the case of we have investigated this circuit model to ten coupled oscillators as random network model without hub.

For the future work, we would like to consider the influence of the small mismatch for synchronization state of more large scale networks or different types of network.

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