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Comparison of Synchronization Phenomena in Three Network Topologies

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***Abstract*—In this study, we investigate synchronization phenomena of coupled chaotic circuits. The chaotic circuits are combined by resisters on one-dimensional coordinate system. We change the distance between the circuits to adapt the coupling strength. We investigate synchronization phenomena when the distance between the circuits in the group is changed. Also, we measure the phase difference using computer simulations. From the computer simulations, we could make sure of the breakdown of inter-cluster synchronization when the system is changed from the symmetrical system to the asymmetrical system. Additionally, we compare the results of three systems.**

I. Introduction

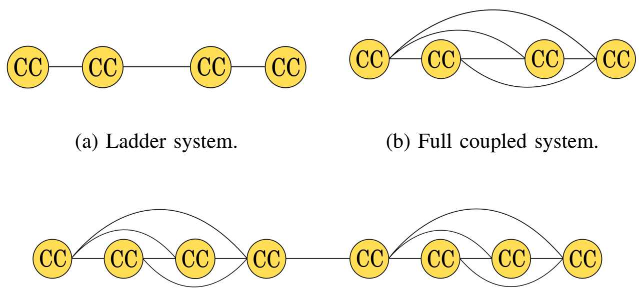
Synchronization phenomenon is one of the typical phenomena observed in nature. Recently, many studies have been investigated synchronization of chaotic circuits [1]~[5]. It is focused how the differences of the network structure impact on the whole circuits. Additionally, it is applicable to the fields of medical science and biology and so on.

In our research group, we have observed the synchronization phenomena from symmetrical coupled chaotic circuits and asymmetrical coupled chaotic circuits arranged in one-dimensional coordinate. We used only ladder system. In the ladder system, chaotic circuits are connected to only adjacent circuits. Chaotic circuits were coupled by resister. The number of the circuits was always ten and we investigated symmetrical systems and asymmetrical systems. The distance between the central circuits was fixed. We investigated synchronization phenomena by changing the distance between the circuits. We have made sure of the breakdown of inter-cluster synchronization when the system was changed from the symmetrical system to the asymmetrical system [6].

In this study, we use three systems. The systems are ladder system, bridge system and full coupled system. In the full coupled system, the chaotic circuits are connected to all chaotic circuits. In the bridge system, only the circuits in the group are full coupled. Figure 1 shows each system model. We compare the results of the phase differences in the three systems.

II. Circuit Model

Figure 2 shows the circuit model. This is a chaotic circuit called Nishi-Inaba circuit [7]~[8].



(a) Ladder system. (b) Full coupled system.

(c) Bridge system.

Fig. 1. System models.

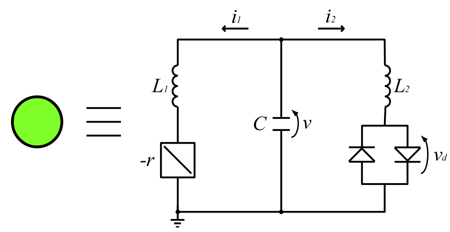


Fig. 2. Circuit model.

The normalized equations of this circuit are obtained as Eq. (1) by changing the variables as below.

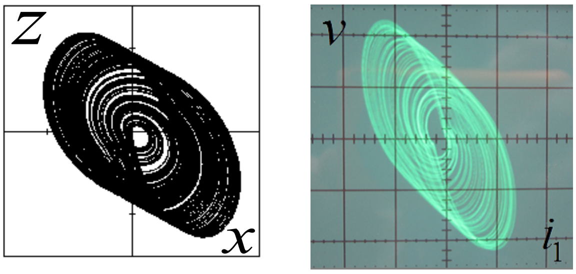
$$i_1 = \sqrt{\frac{C}{L_1}}Vx;\ i_2 = \frac{\sqrt{L_1C}}{L_2}Vy;\ v = Vz;$$
$$r\sqrt{\frac{C}{L_1}} = \alpha;\ \frac{L_1}{L_2} = \beta;\ r_d\frac{\sqrt{L_1C}}{L_2} = \delta;$$
$$t = \sqrt{L_1C}\tau;\quad \text{“}\cdot\text{”} = \frac{d}{d\tau};$$

$$\begin{cases} \dot{x} &= \alpha x + z \\ \dot{y} &= z - f(y) \\ \dot{z} &= -x - \beta y \end{cases} \tag{1}$$

The value of $f(y)$ is described as Eq. (2).

$$f(y) = \frac{\delta}{2}\left(\left|y + \frac{1}{\delta}\right| - \left|y - \frac{1}{\delta}\right|\right) \tag{2}$$

Figure 3 shows the chaotic attractor generated from the circuit by using computer simulation (Fig. 3(a)) and circuit experiment (Fig. 3(b)). For the computer simulation, we set the parameters as $\alpha = 0.460$, $\beta = 3.0$ and $\delta = 470$. For the circuit experiment, the parameters are fixed with $L_1 = 500[mH]$, $L_2 = 200[mH]$, $C = 0.0153[\mu F]$, and $r_d = 1.46[M\Omega]$.



(a) Computer simulation. (b) Circuit experiment.

Fig. 3. Chaotic attractor.

In the ladder system, chaotic circuits are connected to only adjacent circuits.

When chaotic circuits are connected to only adjacent circuits, the circuit equations are shown in Eqs. (4) $\sim$ (6).

Where the parameter γ_{ij} represents the coupling strength between the circuits. The value of γ_{ij} reflects the distance between the circuits in an inverse way, described by the following equation:

$$\gamma_{\{i,j\}} = \frac{g}{(d_{ij})^2}. \tag{3}$$

d_{ij} denotes the Euclidean distance between the i-th circuit and the j-th circuit. The parameter g is coupling coefficient that determines the coupling strengths. In this study, we set the parameter as $g = 1.0 \times 10^{-3}$.

CC_1:

$$\begin{cases} \dot{x}_1 &= \alpha x_1 + z_1 \\ \dot{y}_1 &= z_1 - f(y_1) \\ \dot{z}_1 &= -x_1 - \beta y_1 - \gamma_{\{1,2\}}(z_1 - z_2) \end{cases} \tag{4}$$

CC_n:

$$\begin{cases} \dot{x}_n &= \alpha x_n + z_n \\ \dot{y}_n &= z_n - f(y_n) \\ \dot{z}_n &= -x_n - \beta y_n - \gamma_{\{n,n-1\}}(z_n - z_{n-1}) \\ & \quad -\gamma_{\{n,n+1\}}(z_n - z_{n+1}) \end{cases} \tag{5}$$

CC_N:

$$\begin{cases} \dot{x}_N &= \alpha x_N + z_N \\ \dot{y}_N &= z_N - f(y_N) \\ \dot{z}_N &= -x_N - \beta y_N \\ & \quad -\gamma_{\{N,N-1\}}(z_N - z_{N-1}) \end{cases} \tag{6}$$

In full coupled system, the circuit equation is shown in Eq. (7).

$$\begin{cases} \dot{x}_i &= \alpha x_i + z_i \\ \dot{y}_i &= z_i - f(y_i) \\ \dot{z}_i &= -x_i - \beta y_i - \sum_{j=1}^{N} \gamma_{\{i,j\}}(z_i - z_j) \\ & \quad (i, j = 1, 2, \cdots, N) \end{cases} \tag{7}$$

In bridge system, the circuit equation is shown in Eq. (8). This equation shows the case of using ten circuits.

$$\begin{cases} \dot{x}_i &= \alpha x_i + z_i \\ \dot{y}_i &= z_i - f(y_i) \\ \dot{z}_i &= -x_i - \beta y_i - \Gamma \\ & \quad (i = 1, 2, \cdots, N) \end{cases} \tag{8}$$

$CC_{1\sim4}$:

$$\Gamma = \gamma_{\{n,k\}}(5 \cdot z_n - \sum_{k=1}^{5} z_k) \qquad (n = 1, 2, 3, 4)$$

CC_5:

$$\Gamma = \gamma_{\{5,k\}}(6 \cdot z_5 - \sum_{k=1}^{6} z_k)$$

Due to the symmetry of the system, equation for from CC_6 to CC_{10} is omitted.

III. Simulation Method

We use the three systems arranged in one-dimensional coordinate system. We use ten circuits in computer simulations. We divide into the two symmetrical groups, and there are five circuits in one side of the group.

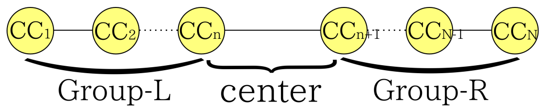


Fig. 4. Network structure.

In the left side and the right side groups, the distances between the circuits are 0.3. The distance between the central circuits is 0.5. The symmetrical network structure is shown in Fig. 5.

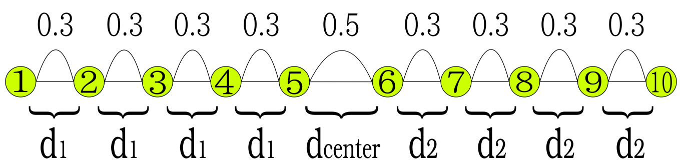


Fig. 5. Symmetrical network structure.

We change the distance between the circuits by changing the coupling strength. We define the distances between the circuits in the left side group as d_1. In the same way, the distances between the circuits in the right side group as d_2. And we define the distance between the central circuits as d_{center}. In this simulation, we fix the values of d_{center} and d_2, and the value of d_1 is changed. The value of d_1 is decreased gradually, and the value of d_1 is changed from 0.3 to 0.1. The asymmetrical network structure is shown in Fig. 6.

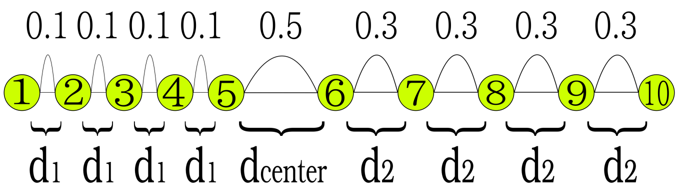


Fig. 6. Asymmetrical network structure.

We measure the phase difference between the circuits using the computer simulation. And we investigate the change in the phase difference when the system is changed from the symmetrical system to the asymmetrical system. Additionally, we compare the results of the three systems.

IV. Simulation Result

Figure 7 shows the graph of comparison of the results in each system. This figure is focused on only the phase difference of the central circuits.

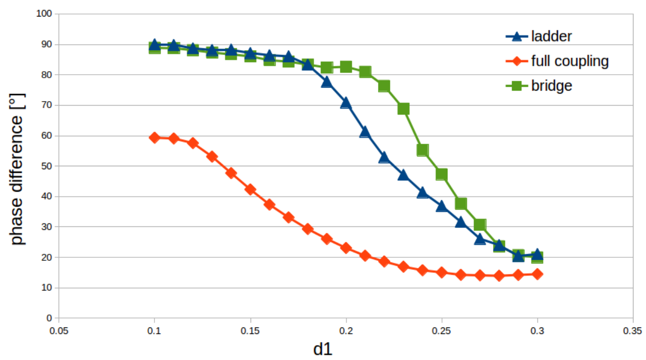


Fig. 7. Comparison of the results in each system (between the central circuits).

From the simulation result, in the ladder system, the phase difference is increasing gradually. And the central circuits become asynchronous around $d_1 = 0.19$. In the bridge system, the phase difference is increasing gradually in the same way as ladder system. However the central circuits in the bridge system becomes asynchronous around $d_1 = 0.22$. Although the number of coupling of the bridge system is heavier than the ladder system, the bridge system become faster asynchronous than the ladder system. In the bridge system, only the circuits in the group are full coupled. We consider that the bridge system becomes asynchronous faster than the ladder system, because the coupling in the group becomes stronger. In the full coupled system, the central circuits do not become asynchronous for large number of the coupling.

From this result, we can see that the strength of synchronization is not affected by the number of the coupling of the systems.

V. Conclusions

In this study, we have investigated the synchronization phenomena in coupled chaotic circuits networks. We also investigated the phase difference in the symmetrical network system and the asymmetrical network system. Additionally, we have compared the results of three systems. From the computer simulation, we have confirmed the similar results in all of the system. Additionally, we have obtained very interesting results to compare the central circuits of the three systems.

For the future work, we would like to confirm the same results by using the circuit experiments.

Acknowledgment

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