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Synchronization of Ring Combination of van der Pol Oscillators with Different Frequencies

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1. Introduction

In this study, we investigate synchronization phenomena observed from coupled van der Pol oscillators with different frequencies. When the frequencies of three oscillators are the same, we observe three-phase synchronization. As increasing the difference of the frequencies, the three-phase synchronization breaks down and becomes asynchronous state. By computer simulations, we investigate how the synchronization breaks down.

2. System model

Figure 1 shows the system model. Three van der Pol oscillators are coupled as a ring by resistors through inductors.

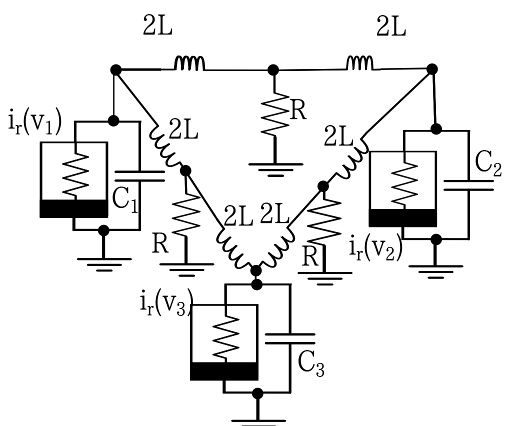

Figure 1: System model.

The circuit equations are described by the following equations.

$$
\begin{cases}
\dot{x}_1 = \epsilon(x_1 - \frac{x_1^3}{3}) - (y_1 + z_1) \\
\dot{x}_2 = \epsilon\omega_1^2(x_2 - \frac{x_2^3}{3}) - \omega_1^2(y_2 + z_2) \\
\dot{x}_3 = \epsilon\omega_2^2(x_3 - \frac{x_3^3}{3}) - \omega_2^2(y_3 + z_3) \\
\dot{y}_i = \frac{y_i}{2} - \frac{\beta}{2}(y_i + z_{i+1}) \\
\dot{z}_i = \frac{z_i}{2} - \frac{\beta}{2}(z_i + y_{i-1}) \\
\qquad (i = 1, 2, 3)
\end{cases}
\qquad (1)
$$

where the parameter β represents the coupling strength between the circuits and the parameter ϵ represents nolinearity of the oscillators. These parameters are fixed as ϵ =0.1000 and β=0.1500. The parameters ω_1 and ω_2 represent the difference of the frequencies. If $\omega_1 = \omega_2$ =1.0, the three oscillators have the same frequency and are synchronized in three-phase.

3. Simulation results

In this study, we carry out computer simulations for $\omega_1 = 1.001 \sim 1.050$ and $\omega_2 = 1.002 \sim 1.100$. Figure 2 shows some examples of the simulation results. For small difference of frequencies, the oscillators exhibit three-phase synchronization as shown in Fig. 2(a). Next, we increase the frequency difference until they become asynchronous. Figure 2(b) shows slightly disturbed theephase synchronization. Figure 2(c) shows almost asynchronous in the $x_i - x_j$ space, but the time waveforms show that the oscillators still keep the three-phase synchronization. Figure 2(d) shows the asynchronous state.

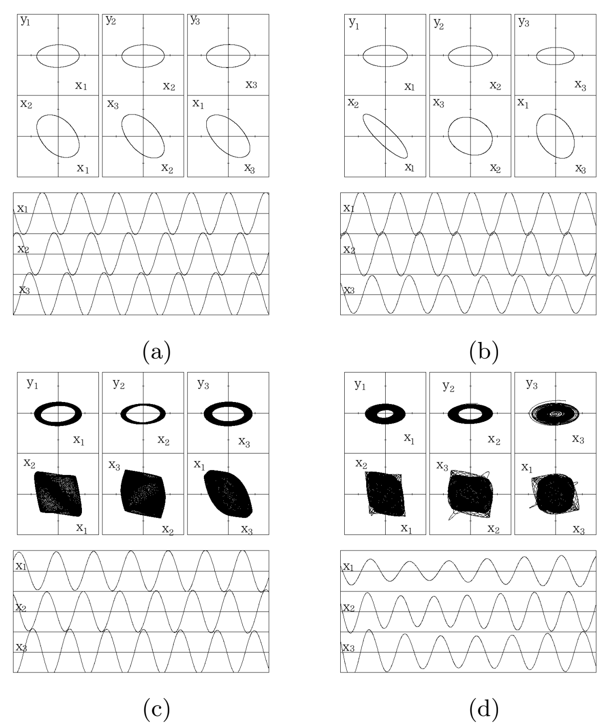

Figure 2: Simulation results.
(a) $\omega_2 = 1.001$. $\omega_3 = 1.002$. (b) $\omega_2 = 1.006$. $\omega_3 = 1.012$.
(c) $\omega_2 = 1.010$. $\omega_3 = 1.020$. (d) $\omega_2 = 1.050$. $\omega_3 = 1.100$.

4. Conclusions

In this study, we researched the synchronization phenomena in coupled three van der Pol oscillators with different frequencies.