Simple Chaotic Oscillator
Composed RC Circuits and Square Wave Generator

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1. Introduction
In this study, we investigate a chaotic oscillator composed RC circuits and square wave generator. By computer simulations, we show the attractor, waveform and bifurcation diagram of proposed circuit.

2. Circuit model
Figure 1 shows the circuit model in this study. Two RC circuits are coupled via comparators of operational amplifiers. The square wave is inputted to the other input terminals of the comparators. The comparators produce the output voltage $V$ which is their power supply voltage, according to the input signals.

![Figure 1: Circuit model.](image)

The exact solutions of normalized circuit equations are described as follows:

$$
\begin{align*}
  x_1 &= \begin{cases} 
    (x_{10} - 1)e^{-\tau} + 1 & (x_2 > \alpha) \\
    (x_{10} + 1)e^{-\tau} - 1 & (x_2 < \alpha) 
  \end{cases} \\
  x_2 &= \begin{cases} 
    (x_{20} - 1)e^{-\tau} + 1 & (x_1 > \alpha) \\
    (x_{20} + 1)e^{-\tau} - 1 & (x_1 < \alpha) 
  \end{cases}
\end{align*}
$$

where $\alpha$ is the parameter corresponding to $V_a$ and $\beta$ is the parameter corresponding to $T$. $x_{10}$ and $x_{20}$ are initial values.

3. Simulation results
We show the chaotic attractor and waveform which are obtained by computer calculation as shown Fig. 2. The parameter of $\alpha$ is 0.06 and $\beta$ is 4.0. The step size of $\tau$ is 0.01. In order to modify an error, we use a method of bisection.

![Figure 2: Simulation results. (a)Attractor. $x_1 - x_2$. (b)Waveform. $x_1 - x_2$.](image)

We observe that the state equations are changed as shown Fig. 2. Red, blue, green and yellow colors denote the case of $x_1$ and $x_2$ are larger than $V_a$, $x_1$ and $x_2$ are smaller than $V_a$, $x_1$ is larger than $V_a$ and $x_2$ is smaller than $V_a$, and $x_1$ is smaller than $V_a$ and $x_2$ is larger than $V_a$ respectively. From these results, there is the periodicity of changing state equation on the proposed circuit.

Figure 3 shows the one parameter bifurcation diagram when $\alpha$ is changed from 0 to 0.1. The vertical axis shows $x_1$ and the horizontal axis shows $\alpha$. Parameter step is 0.01, plotted time is 2000\[s\] and $\beta$ is 4.0. When the plus or minus sign is changed, the value of $x_1$ is plotted on bifurcation diagram.

![Figure 3: Bifurcation diagram.](image)

Bifurcation phenomena of periodic orbits and chaos are observed from Fig. 3.

5. Conclusions
In this study, we have investigated a simple chaotic oscillator composed two RC circuits and square wave generators. We have confirmed that there is the periodicity of changing state equation on the proposed circuit. And, bifurcation phenomena of periodic orbits and chaos are observed.