Synchronization Phenomena of Coupled Rulkov Maps with STDP for Modeling Epilepsy

Naohiro Shibuya†, Charles Unsworth‡, Yoko Uwate† and Yoshifumi Nishio†
†Dept. of Electrical and Electronic Engineering, The University of Tokushima,
2-1 Minami-Josanjima, Tokushima, 770-8506, Japan
Email:{shibuya, uwate, nishio}@ee.tokushima-u.ac.jp
‡Dept. of Engineering Science, The University of Auckland,
70 Symonds Street, Auckland 1001, New Zealand
Email:c.unsworth@auckland.ac.nz

***Abstract*—Epilepsy of the neuropsychiatric disorder is provoked from an imbalance in the long-term potentiation (LTP) versus long-term depression (LTD) of the synapses in the hippocampus. The LTP and LTD are replicated by using the Spike Timing Dependent Plasticity (STDP). Additionally, the spiking activity of the synapses in the hippocampus can be expressed by using the Rulkov Maps. In this paper, we consider a simulation model which is constructed by using Rulkov Maps with STDP. We explore the effect of 5 connective arrangements on spiking activity, as basic simulation for constructing the approximate simulation model of epilepsy. From the result, the tendency of firing has been to improve of synchronization accuracy with the increasing of connective number.**

I. Introduction

Epilepsy is known as one of neuropsychiatric disorder and involved with a large part of the hippocampus. The hippocampus has the lowest seizure threshold in the brain, therefore it has indicated the beginning of most epilepsy seizures. Moreover, the seizure-related neuronal electrical activity has feature of synchrony arising regularly [1],[2]. In the normal status, some neurons suppress the abnormal firing. If the neuronal firing property of neurons or potentiation and depression balance are altered slightly, the supranormal excitability is diffused and lead to seizure. We consider that the each potentiation and the depression corresponds approximately to each long-term potentiation (LTP) and long-term depression (LTD).

In this study, we apply Spike Timing Dependent Plasticity (STDP) to Rulkov Maps for constructing the small-scale simulation models. STDP is a generic model used to replicate the LTP and the LTD [3],[4], and Rulkov Map produces two-dimensional spiking-bursting behavior like real biological neurons [5]-[7]. By incorporating the STDP into the original Rulkov Map, we can explore the synchronous behavior like real biological spiking activity. We explore the effect of 5 connective arrangements on spiking activity, as basic simulation for constructing the approximate simulation model of epilepsy. Moreover, we investigate a relationship between synchronous firing and LTP, LTD.

II. Spike Timing Dependent Plasticity

STDP is a temporally asymmetric form of Hebbian learning induced by tight temporal correlations between the spikes of pre- and postsynaptic neurons. STDP provokes the LTP of the synapses, if the presynaptic spike arrival a few milliseconds before postsynaptic spikes. Whereas, it provokes the LTD of the same synapse, if presynaptic spike arrival after postsynaptic spikes.

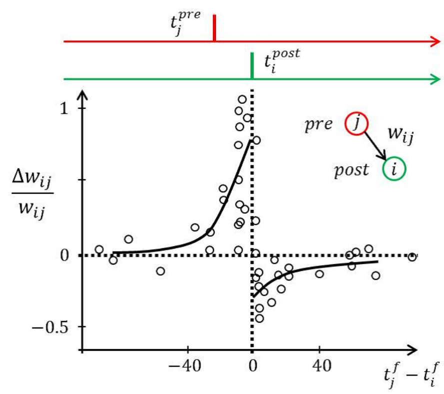


Fig. 1. The STDP function of changing synaptic connection.

The weight change Δw_{ij} depends on the relative timing between presynaptic spike arrivals and postsynaptic spikes. The total weight change Δw_{ij} induced by a simulation protocol with pairs of pre- and postsynaptic spikes is shown as following.

$$\Delta w_{ij} = \sum_{f=1}^{N} W(t_j^f - t_i^f) \tag{1}$$

where $W(x)$ denotes one of the STDP functions in Fig. 1. The method to choose for the STDP function $W(x)$ is shown as following.

$$W(x) = \begin{cases} A_+ exp(-x/\tau_+) & (x < 0) \\ -A_- exp(x/\tau_-) & (x > 0) \end{cases} \tag{2}$$

which has been used in fits to experimental data and models. The parameters A_+ and A_- may depend on the current value of the synaptic weight w_{ij}. In this article, the parameter A_+ and A_- are fixed 0.05, and the time constants are on the order of $\tau_+ = 10ms$ and $\tau_- = 10ms$.

Moreover, the STDP function as in Eq. (2) can be implemented in an on-line update rule using the following equation.

$$\frac{dw_{ij}}{dt} = A_+(w_{ij})x_j(t)\sum_f \delta(t - t_i^f) \\ -A_-(w_{ij})y_i(t)\sum_f \delta(t - t_j^f) \qquad (3)$$

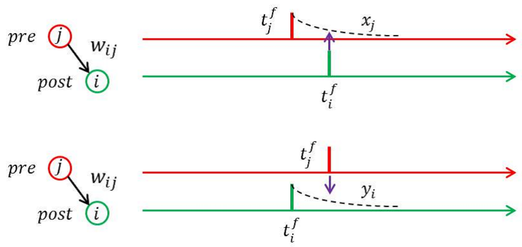


Fig. 2. Top: A presynaptic spike trace $x_j(t)$ with postsynaptic spike. Bottom: A postsynaptic spike trace $y_i(t)$ with presynaptic spike.

III. Rulkov Map

In recent years, a simple model which replicates the dynamics of spiking and spiking-bursting activity of real biological neurons has proposed by N. F. Rulkov. The model is a two-dimensional map that produces chaotic spiking-bursting neural behavior. It is demonstrated that the results of this model are in agreement with the synchronization of chaotic spiking-bursting behavior experimentally found in real biological neurons. The expressions of the Rulkov map are shown as following.

$$\begin{aligned} x_{n+1} &= f(x_n, x_{n-1}, y_n) \\ y_{n+1} &= y_n - \mu(x_n + 1) + \mu\sigma + \mu\sigma_n \end{aligned} \qquad (4)$$

where x is the fast and y is the slow dynamical variables. α and μ, σ shows the parameters of the maps. In this paper, we set parameter α with arbitrary value, and μ, σ are fixed to 0.002, 0.24 each. Here, these parameters are the control the dynamics parameters and the behavior shows the typical of the neurons. Specifically, the amplitude of waveform is changing by parameter α. The nonlinear function $f(x_n,\ x_{n-1},\ y_n)$ is shown as following:

$$f(x_n, x_{n-1}, y_n) = \begin{cases} \alpha/(1 - x_n + y_n), & (x_n \leq 0) \\ \alpha + y_n, & (0 < x_n < \alpha + y_n \ and \ x_{n-1} \leq 0) \\ -1, & (x_n \geq \alpha + y_n \ or \ x_{n-1} > 0) \end{cases} \qquad (5)$$

IV. Coupled Rulcov Maps with STDP

First, we explain about the coupled Rulkov Map. The equation of the coupled Rulkov Maps are shown as following.

$$\begin{aligned} x_{m,n+1} &= f(x_{m,n}, x_{m,n-1}, y_{m,n}) \\ &\quad + \frac{1}{2}w_{ij}(x_{m+1,n} - 2x_{m,n} + x_{m-1,n}) \\ y_{m,n+1} &= y_{m,n} - \mu(x_{m,n} + 1) + \mu\sigma + \mu\sigma_{m,n} \\ &\quad + \frac{1}{2}w_{ij}(x_{m+1,n} - 2x_{m,n} + x_{m-1,n}) \end{aligned} \qquad (6)$$

where parameter w_{ij} shows the coupling weight of the connection between the maps. Each parameters of x_1, x_2 sets individually.

The coupling weights w_{ij} are updated by STDP function. If STDP provokes the LTP, the coupling weights are updated to positive direction. Whereas, if it provokes LTD, the coupling weights are updated to negative direction. We consider that the Rulkov Maps show in-phase/reverse-phase synchronization by changing the coupling weight.

A. 2 Coupled Maps

In this section, we observe fundamental synchronization phenomena of the coupled Rulkov Map. We consider two coupled Rulkov Maps with STDP function as shown in Fig. 3. Additionally, we set the parameter $\alpha = 10, 15$, and explore the spiking activity and coupling weight, if the coupling direction is changed. The spiking activity of coupled Rulkov Maps and coupling weights are shown in Fig. 4.

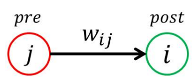
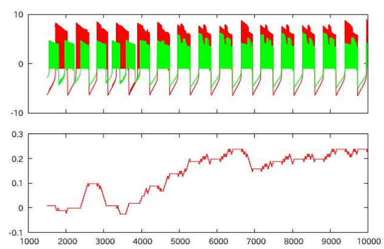
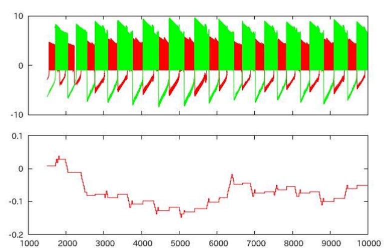


Fig. 3. 2 coupled Rulkov Maps with STDP.



(a) $\alpha_1 = 15$, $\alpha_2 = 10$.



(b) $\alpha_1 = 10$, $\alpha_2 = 15$.

Fig. 4. The spiking activity of 2 coupled Rulkov Maps and the coupling weights. (a) In-phase synchronization. (b) Reverse phase synchronization.

From the result of Fig. 4, we can see that the synchronization phenomenon shows inverting phase by changing the coupled direction. Additionally, we can observe the in-phase/reverse-phase synchronization if the coupling weight is set to positive/negative value. However, the coupling weight is not symmetric about the x-axis. Therefore, we consider that the some neurons are able to interconnect.

B. 4 Coupled Maps with 5 Connective Arrangements

In this section, we consider the 4 coupled Rulkov Maps. We have defined these connective arrangements to be: ‘linear unidirectional’, ‘linear bidirectional’, ‘box unidirectional’, ‘box bidirectional’, and ‘all-to-all’ [8]. Each configuration is shown in Fig. 5. We explore the effect of 5 connective arrangements on neuronal dynamics of spiking activity. Moreover, we investigate an relationship between synchronous firing and LTP, LTD.

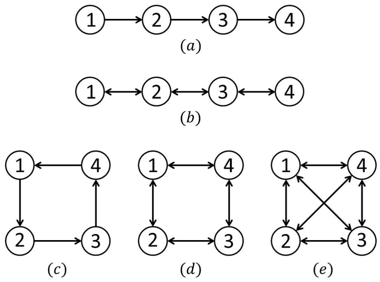


Fig. 5. Connective arrangements of 4 coupled Rulkov Maps. (a) Linear unidirectional. (b) Linear bidirectional. (c) Box unidirectional. (d) Box bidirectional. (e) All-to-all.

The parameter $(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$ are fixed $(15,\ 10,\ 20,\ 10)$. Moreover, we define the synchronization probability is shown as follows.

$$I = \frac{Y}{X} \tag{7}$$

where X is the number of burst waveform, and Y is the number of synchrony burst waveform.

The simulation results of spiking activity (from 0 to 30000) and the synchronization probability (from 0 to 15000, from 15001 to 30000) are shown in Fig. 6. The ‘linear bidirectional’ model shows different clustering phenomena as the ‘linear unidirectional’ by interconnecting the neurons. The ‘box bidirectional’ model also shows different clustering phenomena as the ‘box unidirectional’ by interconnecting the neurons. From these results, we consider that the effects of interconnecting of neurons are making stable waveform, and improving the synchronization accuracy. Finally, the ‘all-to-all’ model shows high accuracy in-phase/reverse-phase synchronization with duration. Overall, we can see that the tendency of firing has been to improve of synchronization accuracy with the increasing of connective number.

V. Conclusion

In this paper, we considered a simulation model which is constructed by using Rulkov Maps with STDP. We have explored the effect of 5 connective arrangements on spiking activity, as basic simulation for constructing the approximate simulation model of epilepsy. Moreover, we investigated an relationship between synchronous firing and LTP, LTD. From the result, if STDP provokes the LTP, the coupling weights are updated to positive direction. Whereas, if it provokes LTD, the coupling weights are updated to negative direction. Overall, the tendency of firing has been to improve of synchronization accuracy with the increasing of connective number.

References

[1] P. A. Schwartzkroin, “Role of the hippocampus in epilepsy,” *Hippocampus*, vol. 4, pp. 239-242, 1994.

[2] K. Nakashima, H. Hayashi, O. Shimizu and S. Ishizuka, “Long-term change in synaptic transmission in CA3 circuits followed by spontaneous rhythmic activity in rat hippocampal slices,” *Neuroscience Research*, vol. 40, pp. 325-336, 2001.

[3] A. Morrison, M. Diesmann and W. Gerstner, “Phenomenological models of synaptic plasticity based on spike timing,” *Biological Cybernetics*, vol. 98, pp. 459-478, 2008.

[4] J. P. Pfister, W. Gerstner, “Triplets of spikes in a model of spike timing-dependent plasticity,” *Journal of Neuroscience*, vol. 26, pp. 9673-9682, 2006.

[5] N. F. Rulkov, I. Timofeev and M. Bazhenov, “Oscillations in Large-Scale Cortical Networks: Map-Based Model,” *Journal of Computational Neuroscience*, vol. 17, pp. 203-223, 2004.

[6] A. L. Shilnikov, N. F. Rulkov, “Origin of chaos in a Two-dimensional Map Modeling Spiking-Bursting Neural Activity,” *Int. J. Bifurcation and Chaos*, vol. 13, pp. 3325-3340, 2003.

[7] N.F. Rulkov, “Modeling of Spiking-Bursting Neural Behavior using Two-dimensional Map,” *Physical Review E*, vol. 65, 041922, 2002.

[8] D. Cumin, C. P. Unsworth, “Generalising the Kuromoto Model for the study of Neuronal Synchronisation in the Brain,” *Phycsica D: Nonlinear Phenomena*, vol. 226, pp. 181-196, 2007.

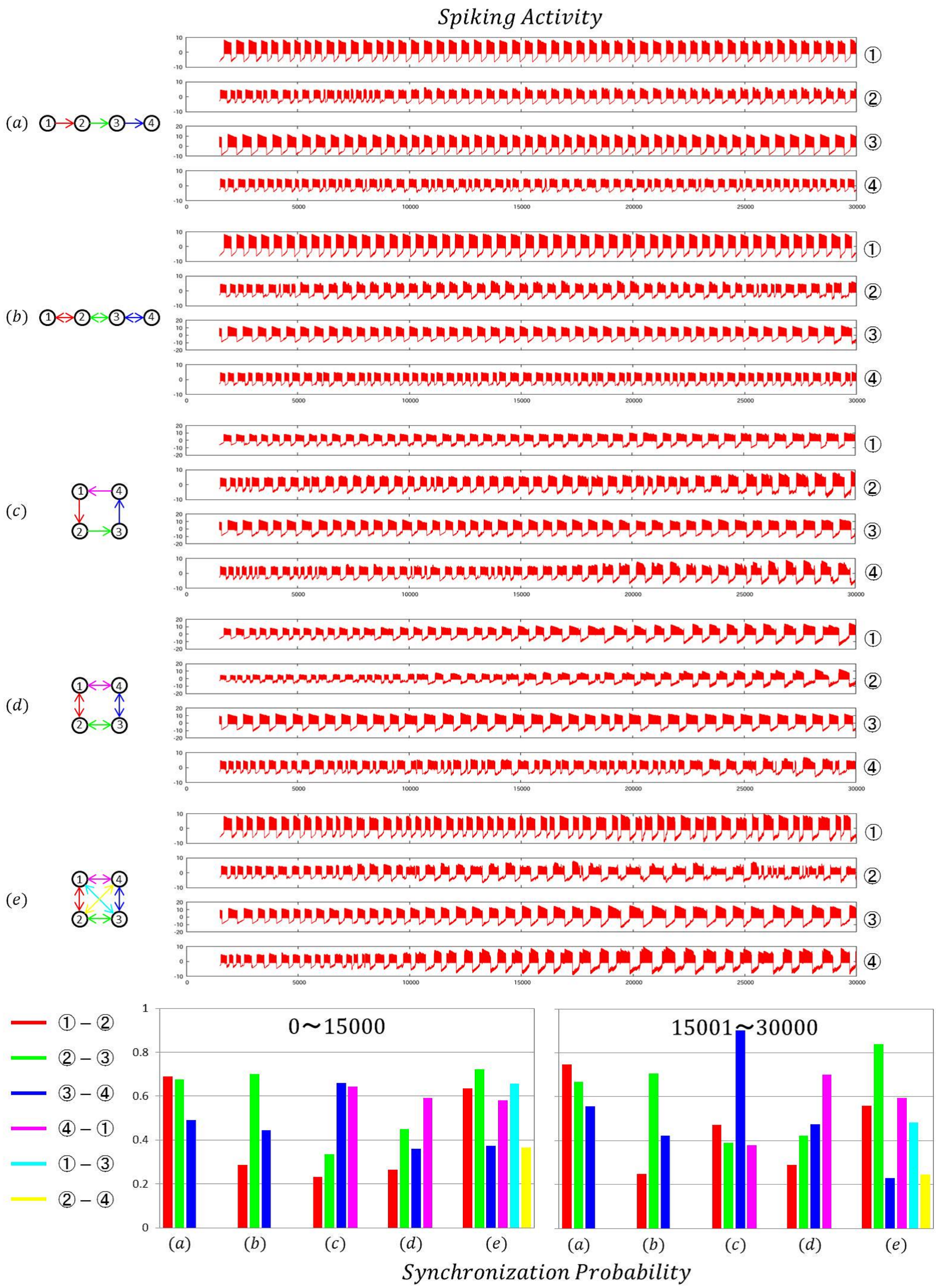


Fig. 6. The spiking activity of 4 coupled Rulkov Maps and the synchronization probability. $\alpha_1 = 15$, $\alpha_2 = 10$, $\alpha_3 = 20$, $\alpha_4 = 10$.