Synchronization Phenomena of Coupled Chaotic Circuit Network with Bridge

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***Abstract*— In this study, we investigate synchronization phenomena of coupled chaotic circuit network with the bridge. By computer simulations, synchronization of the bridge on the network is easy to break down and clustering of circuits is occurred from the bridge. Moreover, we statistically investigate synchronous rate.**

I. Introduction

Recently, engineering applications of chaos have received a lot of attention by many researchers. For example, communication systems of chaos, control of chaos, synchronization of chaos and so on. Especially, synchronization of chaos is very interesting phenomena that chaotic elements synchronize in spite of different initial values[1]. Additionally, coupled systems of chaotic elements generate many kinds of complexity phenomena such as spatiotemporal intermittency[2], clustering[3], and so on. Coupled Map Lattice (CML) and Globally Coupled Map (GCM) proposed by Kaneko are very simple and carried out for discrete–time mathematical model. However, many of nonlinear phenomena generated in nature would be not so simple. Therefore, it is important to investigate the complex phenomena observed in continuous–time real physical system such as electrical circuits[4]–[6].

In this study, we investigate synchronization phenomena in coupled chaotic circuit network with the bridge. The Bridge is the edge which provide the only route between two nodes. In order to analyze complex phenomena of the bridge on the network, each node is applied chaotic circuits and connected via registers. By computer simulations, we observe synchronization phenomena between circuits. Moreover, we statistically investigate synchronous rate.

II. Network model

Figure 1 shows chaotic circuit which is three-dimensional autonomous circuit proposed by Mori et al.[7][8]. A Proposed network model is shown in Fig. 2. In this study, chaotic circuits are applied to nodes of the network and are used resistors as coupling elements. In this model, the bridge is the resistor between CC3 and CC4.

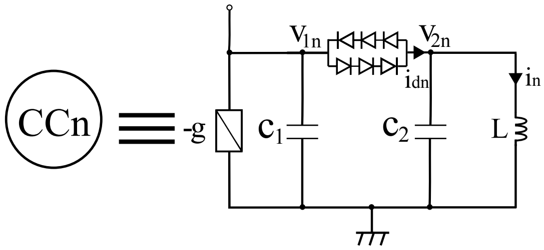

Fig. 1. Chaotic circuit.

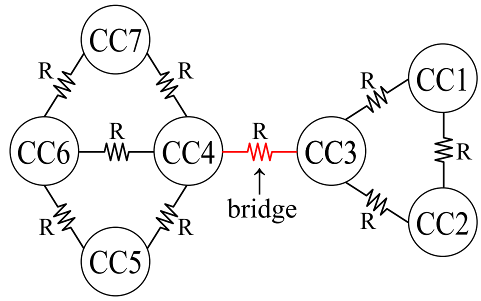

Fig. 2. Model of coupled chaotic circuits network with bridge.

First, the circuit equations are given as follows:

$$\begin{cases} L\dfrac{di_n}{dt} = v_{2n} \\ C_1\dfrac{dv_{1n}}{dt} = gv_{1n} - i_{dn} - \dfrac{1}{R}\displaystyle\sum_{k\in C_n}(v_{1n} - v_{1k}) \\ C_2\dfrac{dv_{2n}}{dt} = -i_n + i_{dn}, \end{cases} \tag{1}$$

$$(n = 1, 2, 3, ..., 7),$$

where C_n is set of nodes which are connected to CCn. We approximate the $i-v$ characteristics of the nonlinear resistors consisting of the diodes by the following 3-segment piecewise-linear function,

$$i_{dn} = \begin{cases} G_d(v_{1n} - v_{2n} - a) & (v_{1n} - v_{2n} > a) \\ 0 & (|v_{1n} - v_{2n}| \le a) \\ G_d(v_{1n} - v_{2n} + a) & (v_{1n} - v_{2n} < -a). \end{cases} \tag{2}$$

By using the parameters and variables as follows:

$$
\begin{aligned}
&i_n = \sqrt{\frac{C_2}{L}}ax_n,\ v_{1n} = ay_n,\ v_{2n} = az_n \\
&t = \sqrt{LC_2}\tau,\ \text{“}\cdot\text{”} = \frac{d}{d\tau},\ \alpha = \frac{C_2}{C_1} \\
&\beta = \sqrt{\frac{L}{C_2}}G_d,\ \gamma = \sqrt{\frac{L}{C_2}}g,\ \delta = \frac{1}{R}\sqrt{\frac{L}{C_2}},
\end{aligned}
\tag{3}
$$

the normalized circuit equations are given as follows:

$$
\begin{cases}
\dot{x_n} = z_n \\
\dot{y_n} = \alpha\gamma y_n - \alpha\beta f(y_n - z_n) - \alpha\delta \displaystyle\sum_{k\in C_n}(y_n - y_k) \\
\dot{z_n} = \beta f(y_n - z_n) - x_n,
\end{cases}
\tag{4}
$$

where the nonlinear function corresponding to the characteristics of the nonlinear resistor of the diodes and are described as follows:

$$
f(y_n - z_n) = \begin{cases}
y_n - z_n - 1 & (y_n - z_n > 1) \\
0 & (|y_n - z_n| \le 1) \\
y_n - z_n + 1 & (y_n - z_n < -1).
\end{cases}
\tag{5}
$$

This circuit generates chaotic attractor as shown in Fig. 3.

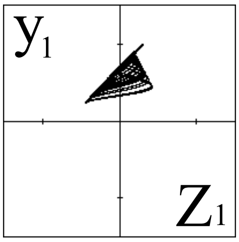

Fig. 3. Example of chaotic attractor. $\alpha = 0.4$, $\beta = 0.5$ and $\gamma = 20$.

III. Simulation Result

In this study, we fix the parameters as $\alpha = 0.4$, $\beta = 0.5$, $\gamma = 20$ and δ on all circuits. We focus on the coupling strength δ. Each circuit is given different initial values each other. Figure 4 shows the computer simulation result in the case of $\delta = 0.25$. Also, the vertical axes are the differences between the voltage of the two chaotic circuits. Namely, if the two chaotic circuits synchronize, the value of the graph should be almost zero like $y_1 - y_2$. In Fig. 4, synchronization of the bridge ($y_3 - y_4$) is easy to break down compared with other circuit combinations and clustering of circuits is occurred from the bridge.

Moreover, in order to analyze synchronization, we define the synchronization as following equation,

$$
|y_n - y_k| < 0.01,
\tag{6}
$$

where n is the number of circuits and k is circuits which connected circuit n. Figure 5 shows statistically investigated result of synchronous rate. Synchronous rate of Fig. 5 corresponds to Fig. 4. Also, iteration time is fixed with 10,000,000. In Fig. 5, we confirmed that CC1 and CC2 synchronized almost completely. On the other hands, the other combinations show low synchronous rate. In addition, the bridge (CC3 and CC4) is lowest synchronous rate of all combinations. This result depending on the initial values can be observed since each circuit generates asymmetric attractor.

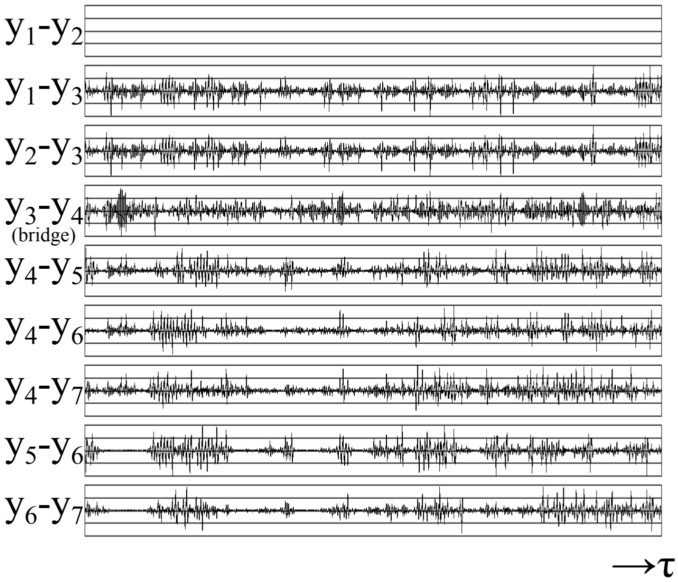

Fig. 4. Computer simulation result. $\delta = 0.25$.

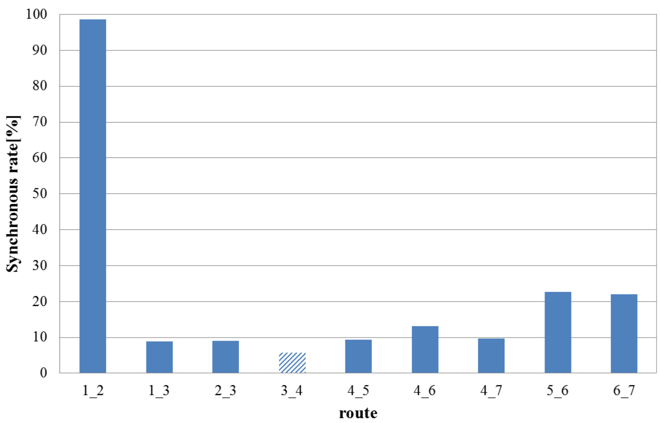

Fig. 5. Synchronous rate. $\delta = 0.25$.

IV. Conclusion

In this study, we have investigated influence of the bridge on the network via synchronization phenomena of coupled chaotic circuits. By computer simulations, synchronization of the bridge is easy to break down and clustering of circuits is occurred from the bridge. In our future works, we investigate the other networks with the bridge and carry out circuit experiment.

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