Synchronizations Phenomena in Two Coupled Chaotic Circuits Containing Time Delay Coupling

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***Abstract*— Studies on chaos synchronization in coupled chaotic circuits are extensively carried out in various fields. Moreover, interesting phenomena were confirmed in a system including a time delay. In this study, we investigate two Nishio-Inaba chaotic circuits coupled via resistor containing time delay coupling. Interesting synchronization phenomena can be confirmed by circuit experiments and computer simulations.**

I. Introduction

Synchronization phenomena in complex systems are very good models to describe various higher-dimensional nonlinear phenomena in the field of natural science. Recently, many studies have been investigated synchronization of chaotic circuits. It is applied in the field of engineering, physics and biology and so on [1]-[4]. Moreover,, interesting phenomena were confirmed in a system including a time delay [5]. We consider that it is very important to investigate the phenomena related with chaos synchronization to realize future engineering application utilizing chaos.

In this study, we consider two Nishio-Inaba chaotic circuits [6]-[8] coupled via resistor containing time delay coupling. Coupling this system is controlled by the switch. In additional, the switches connection is controlled by the amplitude. Additionally, this switching operation is included delay, and various phenomena were occurred by varying connection time. Therefore, we investigate the synchronization phenomena by using the computer calculated and circuit experiments.

II. Circuit Model

Figure 1 shows the circuit model. In the circuit, two Nishio-Inaba circuits are coupled via resistor R and switch SW. The circuit consists of a negative resistance, a nonlinear resistance consisting of two diodes, capacitor and two inductors. Furthermore, SW is the switch controlled by the amplitude, switch is connected to the resistor R in the case of amplitude is larger than the threshold, and connect time is represented by T_c. Also, this switching operation is included delay, and delay time is represented by T_d. Figure 2 shows the switching operation. Voltage reaches the threshold at a certain time, the switch is not connected to the resistor immediately, which is connected for T_c seconds after T_d seconds. Namely, v_{1n} reaches V_{th}, two chaotic circuits coupled via resistor R is connected for T_c seconds after T_d seconds. Figure 3 shows the circuit realization. This circuit is consisted two Monostable Multivibrators. Monostable Multivibrator 1 outputs a pulse at regular intervals in reaction to the rising edge of the input pulse, and Monostable Multivibrator 2 outputs a pulse at regular intervals in reaction to the falling edge of the input pulse. Pulse width of Monostable Multivibrator 1 is corresponding to the delay time, and pulse width of Monostable Multivibrator 2 is corresponding to the connection time.

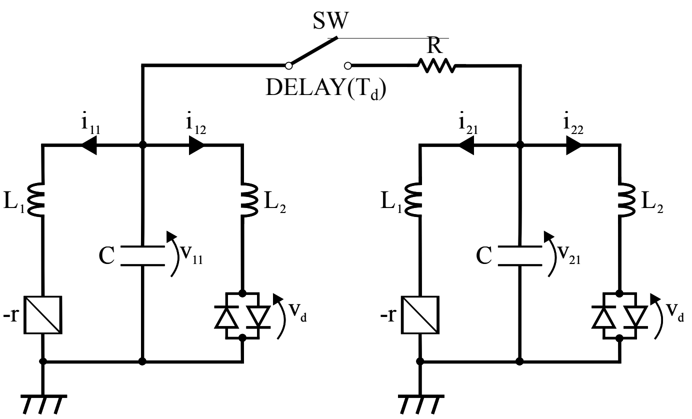

Fig. 1. Circuit model.

By using the variables such that

$$\begin{cases} i_{n1} = \sqrt{\dfrac{C}{L_1}} V x_n, \quad i_{n2} = \dfrac{\sqrt{L_1 C}}{L_2} V y_n, \quad v_{n1} = V z_n \\ \alpha = r\sqrt{\dfrac{C}{L_1}}, \quad \beta = \dfrac{L_1}{L_2}, \quad \delta = r_d \dfrac{\sqrt{L_1 C}}{L_2}, \\ \gamma = \dfrac{1}{R}\sqrt{\dfrac{L_1}{C}}, \quad t = \sqrt{L_1 C_2}\tau, \quad \text{" . "} = \dfrac{d}{d\tau}. \end{cases} \tag{1}$$

That the equation (1) is normalized as

$$\begin{cases} \dot{x_n} = \alpha x_n + z_n \\ \dot{y_n} = z_n - f(y_n) \\ \dot{z_1} = -x_1 - \beta y_n - \gamma(z_1 - z_2) \\ \dot{z_2} = -x_2 - \beta y_n + \gamma(z_1 - z_2) \end{cases} \tag{2}$$

$$(n = 1, 2),$$

The nonlinear function $f(y_n)$ corresponds to the $I - V$ characteristics of the nonlinear resistors consisting of the diodes and are assumed to be described as follows;

$$f(y_n) = \frac{\delta}{2}\left(\left|y_n + \frac{1}{\delta}\right| - \left|y_n - \frac{1}{\delta}\right|\right) \tag{3}$$

$$(n = 1, 2).$$

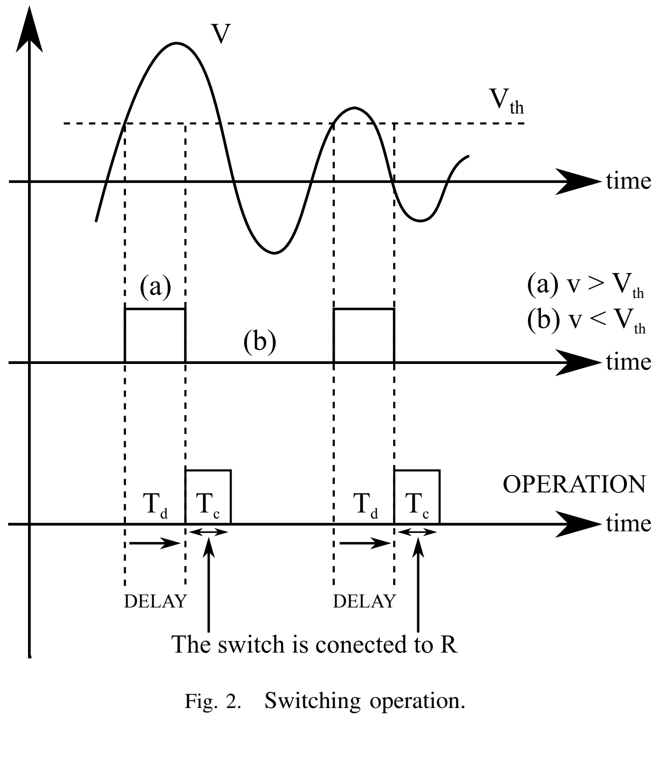

Fig. 2. Switching operation.

III. Synchronization Phenomena

We investigate the phenomenon for the case of varying T_c in the circuit of Fig. 1.

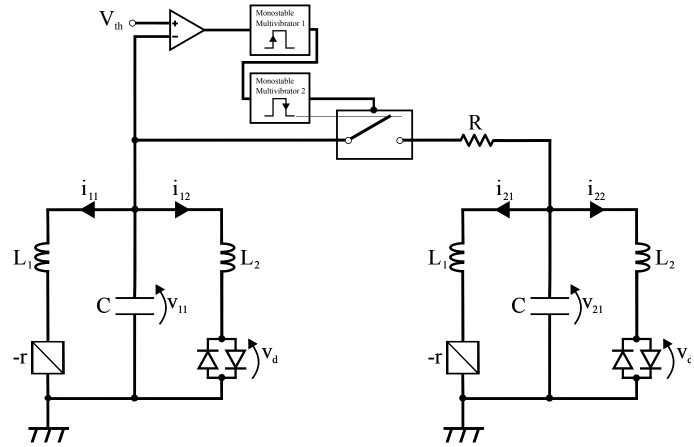

Fig. 3. Circuit realization.

A. Circuit Experiments Results

Figure 4 is obtained by cutting out a part of the state to varies continuously.

Figure 4 shows circuit experiments results for the case of varying the T_c and the parameters are fixed with $L_1 = 500[mH]$, $L_2 = 200[mH]$, $C = 0.0153[\mu F]$, $r_d = 1.46[M\Omega]$, $R = 43.2[k\Omega]$ and $V_{th} = 5.50[V]$. In Fig. 4(a), two circuits are asynchronous including the in-phase synchronization and anti-phase synchronization. However, in Figs. 4(b) and 4(c), two circuits are synchronized in in-phase. As a result, we confirmed two circuits are synchronized with the increase of T_c.

B. Computer Calculated Results

Figure 5 shows computer calculated results for the case of varying the T_c, and the parameters are fixed with $\alpha = 0.460$, $\beta = 3.0$, $\delta = 470$, $\gamma = 0.132$ and $V_{th} = 0.6$. In Figs. 5(d), 5(e) and 5(f), we confirmed the phase difference between two circuits decrease with the increase of T_c. We can say that this interesting phenomenon can be observed from both computer calculations and circuit experiments.

IV. Conclusions

In this study, we have investigated the synchronization phenomena observed from coupled chaotic circuits containing time delay coupling. We confirmed the phase difference between two circuits decrease with the increase of connection time and phase difference between two circuits switch in asynchronous and in-phase synchronization with the increase of time delay

Investigating the detail of the states and statistical analysis of the observed phenomena are our important future work as well as more detailed explanation of the mechanism of the generations.

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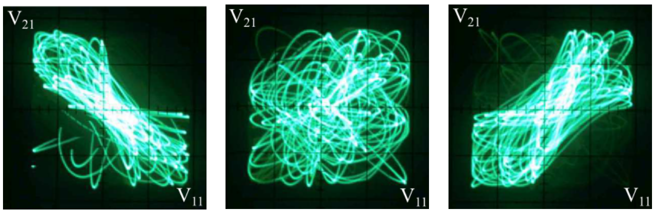
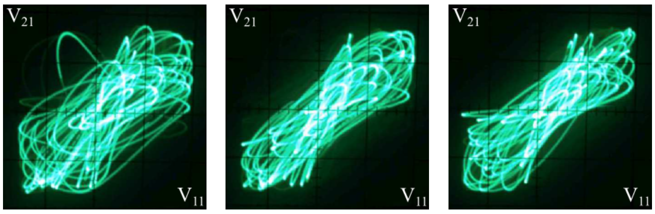
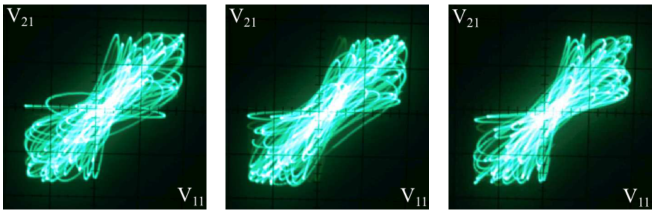


(a) T_d=100μs, T_c=100μs.



(b) T_d=100μs, T_c=330μs.



(c) T_d=100μs, T_c=560μs.

Fig. 4. Phase difference between two circuits for the case of varying the T_c (circuit experiments).

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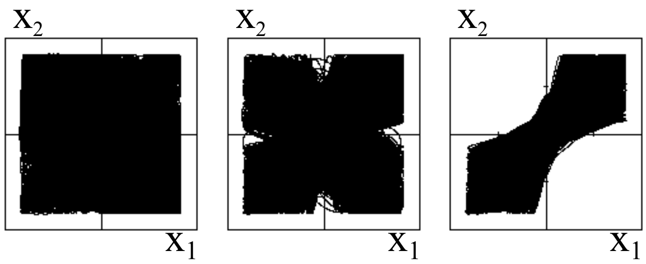
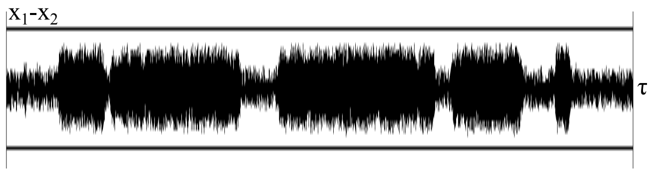
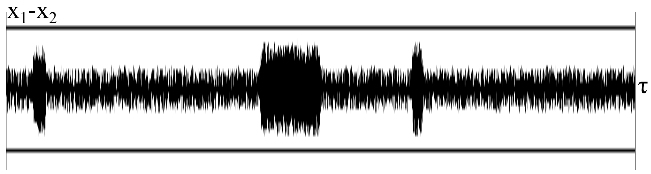


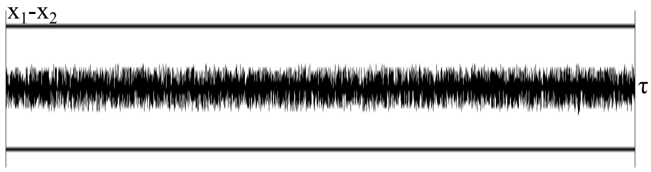
(a) T_d= 50, T_c= 100. (b) T_d= 50, T_c= 200. (c) T_d= 50, T_c= 300.



(d) T_d= 50, T_c= 100.



(e) T_d= 50, T_c= 200.



(f) T_d= 50, T_c= 300.

Fig. 5. State variation for the case of varying the T_c. (a), (b) and (c) phase difference between two circuits (computer simulations). (d), (e) and (f) the value of $x_1 - x_2$ with varying time.