Synchronization Phenomena of Two Chaotic Circuits with Shifting Input Wave

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***Abstract*—In this study, we investigate synchronization phenomena observed in two coupled chaotic oscillators with shifting the input wave. In this circuit system, two chaotic circuits are coupled by one resistor. We investigate the synchronization state when one side's input wave is shifted. We carry out computer calculation to investigate in-phase and anti-phase.**

1. Introduction

Recently, many researchers have shown their interests in chaotic systems [1]-[3]. In particular, chaos synchronization has attracted many researchers' attentions and their mechanism has been gradually made clear. This is because, chaotic systems observing chaos synchronization are good models to explain and describe the higher dimensional nonlinear phenomena in the field of natural science. Further, chaotic behavior has been expected in various fields of research. Hence, chaos has been investigated not only in engineering but also in various fields such as medicine, biology sociology and economics. In the field of electrical and electronic engineering, many applications using chaos has been proposed by many researchers. For example, chaos communications, chaos cryptosystem and chaos neural networks. In order to realize chaotic engineering systems, it is important to understand simple coupled chaos-generating circuits.

In our previous study, a simple chaotic oscillator composed of two RC circuits were proposed [4]. When we changed the parameter, we could observe not only periodic attractor but also chaotic attractor in this simple oscillator. Further, we investigated the chaotic behavior when the number of the coupled RC circuits are changed [5]. Moreover, we found the cross correlation characteristics between neighboring oscillators. In addition, we have investigated synchronization phenomena when two input waves are in-phase and anti-phase [7].

In this study, we investigate synchronization phenomena observed in two coupled chaotic oscillators composed of RC circuits with shifting input wave. Two chaotic oscillators are coupled by one resistor. We carry out computer calculation and investigate chaotic behavior when one side's input wave is shifted.

2. Circuit Model

Figure 1 shows the circuit model in this study. Two chaotic oscillators, which was proposed in [4], are coupled via one resistor. An independent rectangular voltage sources V_{S1} and V_{S2} are connected of each oscillator's two comparators.

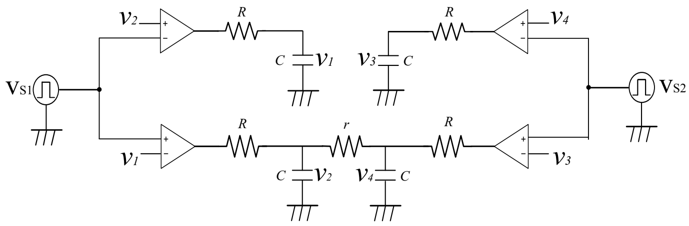

Fig. 1. Circuit model.

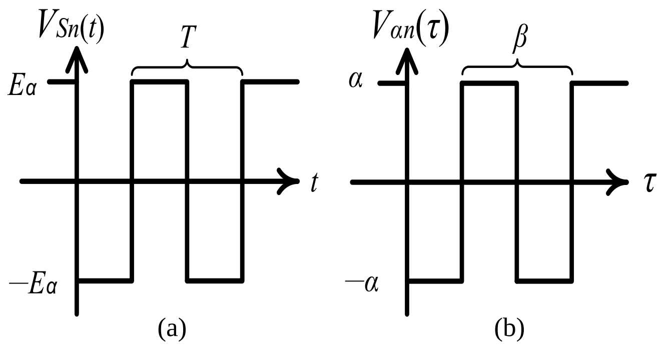

Fig. 2. Rectangular voltage waveform.

Hence, the whole circuit consists of two rectangular voltage sources, four comparators, four resistors, one variable resistor and four capacitors. Figure 2(a) shows the rectangular voltage waveform $V_{Sn}(t)$ $(n = 1, 2)$. $E\alpha$ is the amplitude of the rectangular voltage and T is the period of the waveform. E is the output voltage of the comparators, namely the DC supply voltage of the operational amplifiers.

The circuit equations are described as follows:

$$
\begin{aligned}
RC\frac{dv_1}{dt} &= \begin{cases} -v_1 + E & (v_2 > V_{S1}) \\ -v_1 - E & (v_2 < V_{S1}) \end{cases} \\
RC\frac{dv_2}{dt} &= \begin{cases} -v_2 + \dfrac{R}{r}(v_4 - v_2) - E & (v_1 > V_{S1}) \\ -v_2 + \dfrac{R}{r}(v_4 - v_2) + E & (v_1 < V_{S2}) \end{cases} \\
RC\frac{dv_3}{dt} &= \begin{cases} -v_3 + E & (v_4 > V_{S2}) \\ -v_3 - E & (v_4 < V_{S2}) \end{cases} \\
RC\frac{dv_4}{dt} &= \begin{cases} -v_4 + \dfrac{R}{r}(v_2 - v_4) - E & (v_3 > V_{S2}) \\ -v_4 + \dfrac{R}{r}(v_2 - v_4) + E & (v_3 < V_{S2}) \end{cases}
\end{aligned} \tag{1}
$$

by using the following variables and the parameters,

$$
v_i = Ex_i,\ t = RC\tau,\ T = RC\beta,\ \frac{R}{r} = \gamma,
$$

$$
(i = 1,\ 2,\ 3,\ 4)
$$

we obtain the normalized circuit equations. Because the circuit equations are linear in each region, the rigorous solution of the circuit equations can be derived as follows:

$$
x_1 = \begin{cases} (x_{10} - 1)e^{-\tau} + 1 & (x_2 > V_{\alpha 1}) \\ (x_{10} + 1)e^{-\tau} - 1 & (x_2 < V_{\alpha 1}) \end{cases}
$$

$$
x_2 = \begin{cases} \left(\dfrac{x_{20} + x_{40}}{2} + 1\right)e^{-\tau} \\ \quad + \left(\dfrac{x_{20} - x_{40}}{2}\right)e^{-(1+2\gamma)\tau} - 1 & (x_1 > V_{\alpha 1}) \\ \left(\dfrac{x_{20} + x_{40}}{2} - 1\right)e^{-\tau} \\ \quad + \left(\dfrac{x_{20} - x_{40}}{2}\right)e^{-(1+2\gamma)\tau} + 1 & (x_1 < V_{\alpha 1}) \end{cases}
$$

$$
x_3 = \begin{cases} (x_{30} - 1)e^{-\tau} + 1 & (x_4 > V_{\alpha 2}) \\ (x_{30} + 1)e^{-\tau} - 1 & (x_4 < V_{\alpha 2}) \end{cases}
$$

$$
x_4 = \begin{cases} \left(\dfrac{x_{20} + x_{40}}{2} + 1\right)e^{-\delta\tau} \\ \quad + \left(\dfrac{x_{20} - x_{40}}{2}\right)e^{-(1+2\gamma)\delta\tau} - 1 & (x_3 > V_{\alpha 2}) \\ \left(\dfrac{x_{20} + x_{40}}{2} - 1\right)e^{-\delta\tau} \\ \quad + \left(\dfrac{x_{20} - x_{40}}{2}\right)e^{-(1+2\gamma)\delta\tau} + 1 & (x_3 < V_{\alpha 2}) \end{cases} \tag{2}
$$

where $V_{\alpha n}$ is a parameter corresponding to V_{Sn} $(n = 1, 2)$, β is parameter corresponding to T and γ is a coupling strength. x_{10}, x_{20}, x_{30} and x_{40} are initial values. In this computer calculation, we add the frequency error, in order to set up same condition to the circuit experiment. We carry out computer calculation, the value of the frequency error δ is fixed with 0.05% to remove synchronization isn't due to chaotic phenomena.

3. Simulations

We explain the input wave. The input wave's period β is 10. The step size of τ is 0.01. We define deviation of two input waves as "gap". In this study, we investigate synchronization state when the gap is changed from 0 to 5. For example, Fig. 3 shows two input waves when gap is 2.5.

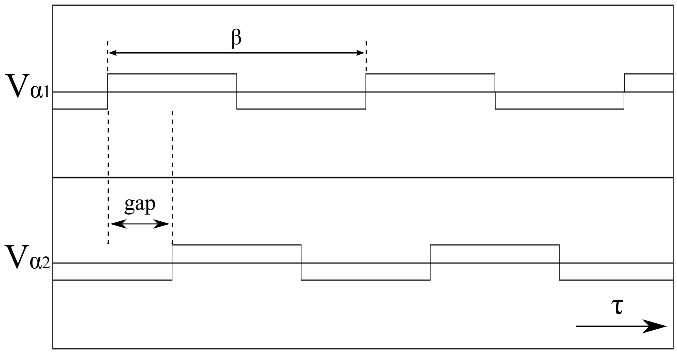

Fig. 3. Two input wave (gap = 2.5).

4. Simulation Results

We show the results of the computer calculations. We use fixed parameters $\alpha = 0.061$ and $\beta = 10$.

Figure 4 show attractor and the phase difference. Left attractors show the left oscillator ($x_1 - x_2$) and center attractors show the right oscillator ($x_3 - x_4$) in Fig. 1. Right attractors show the phase difference between left oscillator and right oscillator ($x_1 - x_3$) in Fig. 1. We can observe the anti-phase synchronization state in Fig. 4(c). This result is caused by two rectangular voltages are anti-phase relations. From these results, we find that it is impossible to observe changing into anti-phase synchronization to in-phase synchronization by shifting input wave when two chaotic oscillators composed RC circuit are coupled via resistor.

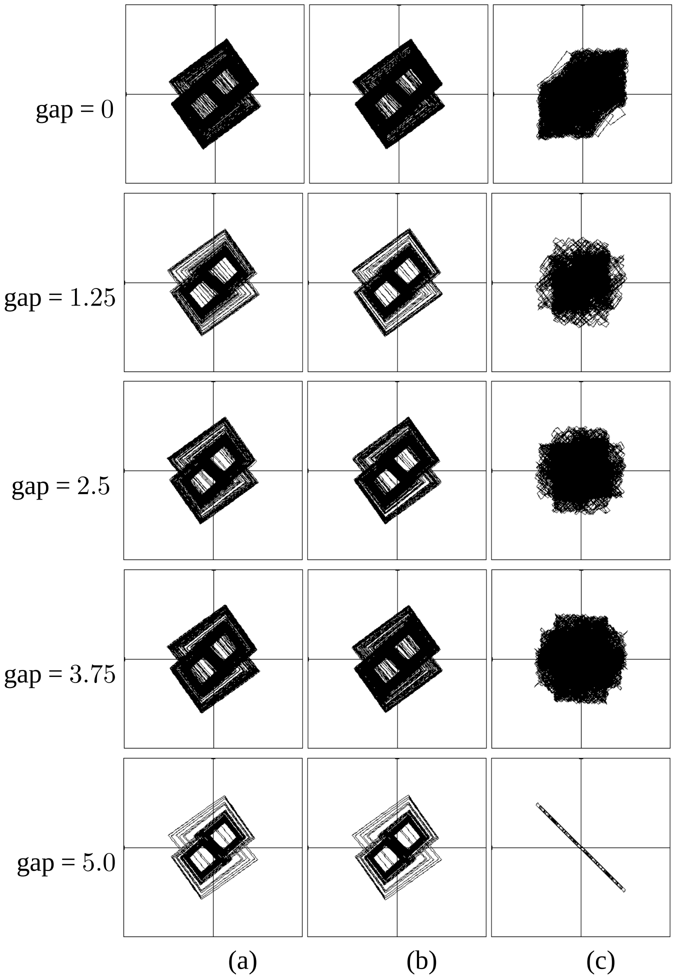

Fig. 4. Chaotic attractor obtained from computer calculation ($\gamma = 0.1$). (a) $x_1 - x_2$, (b) $x_3 - x_4$, and (c) $x_1 - x_3$.

5. Conclusion

In this study, we have investigated synchronization phenomena observed in two coupled chaotic oscillators composed of RC circuits with shifting input wave. Two chaotic oscillators were coupled by one resistor. We carried out computer calculation to investigate.

We confirmed that if the value of coupling strength is small, anti-phase synchronization can be observed. Furthermore, in-phase synchronization can not be observed.

In our future works, we investigate other circuit model composed two coupled chaotic oscillators be coupled by one capacitors. Then, we would like to experiment with electric circuit at the first time. Next, we will investigate larger scale circuits, because this model is very simple structure.

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