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Wave Phenomena Observed from a Ring of Simultaneous Oscillators

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***Abstract*—In this study, we investigate a wave phenomena observed from a ring of simultaneous oscillators with two LC resonators coupled by inductors. We discover the generation of asynchronous oscillations of double-mode and N-phase synchronization by computer simulation.**

I. Introduction

In the natural environment, several synchronization phenomena exist. For example, firefly luminescence, swing of pendulums, cardiac heartbeat, and so on, are well known as synchronization phenomena.

Oscillators containing a nonlinear resistor whose $v-i$ characteristics are described by fifth-power nonlinear characteristics are known to exhibit hard excitation [1][2]. Such an oscillator is often called as hard oscillator or said to have hard nonlinearity.

In 1954, Schaffner reported that an oscillator with two degrees of freedom could oscillate simultaneously at two different frequencies when the nonlinear characteristics are described by a fifth-power polynomial function [3]. Kuramitsu also investigated the simultaneous oscillations for three or more degrees case theoretically and confirmed the generation of simultaneous oscillation with three frequencies by circuit experiments [4]. The simultaneous oscillations are definitely one of the most common nonlinear phenomena observed in various higher-dimensional systems in the natural science fields. However, after their pioneering works, as far as the authors know, there have not been many researches clarifying the basic mechanism of the simultaneous oscillations except [5][6].

Two identical oscillators with hard nonlinearities coupled by an inductor [7] are investigated by Datardina and Linkens. They have confirmed that nonresonant double-mode oscillations, which could not occur for the case of third-power nonlinearity, were stably excited in the coupled system. They have also confirmed that four different modes coexist for some range of parameter values; zero, two single-modes, and a double-mode.

In this study, we investigate a ring of simultaneous oscillators with two LC resonators coupled by inductors. By computer simulations, asynchronous oscillations of double-mode and N-phase are confirmed to be stably generated. [1]

II. Circuit Model

The circuit model is shown in Fig. 1. In the circuit, N simultaneous oscillators with two LC resonators are coupled by inductors L_C as a ring. Each simultaneous oscillator consists of a nonlinear negative resistor, whose $v-i$ characteristics are described by a fifth-power polynomial function as

$$i_R(v) = g_1 v - g_3 v^3 + g_5 v^5 \qquad (g_1,\ g_3,\ g_5 > 0), \tag{1}$$

and two resonators with different natural frequencies ($\sqrt{L_1C_1}$ and $\sqrt{L_2C_2}$). The equations governing these coupled oscillators are described by the following differential equations.

$$\begin{cases} C_1\dfrac{dv_{j1}}{dt} = -i_{j1} - i_{Rj} - i_{Cj} + i_{C,j-1} \\ C_2\dfrac{dv_{j2}}{dt} = -i_{j2} - i_{Rj} - i_{Cj} + i_{C,j-1} \\ L_1\dfrac{di_{j1}}{dt} = v_{j1} \\ L_2\dfrac{di_{j2}}{dt} = v_{j2} \qquad (j = 1, 2, \cdots, N), \end{cases} \tag{2}$$

where $i_{C0} = i_{CN}$. The currents through the coupling inductors i_{Cj} are given as

$$i_{Cj} = \frac{L_1(i_{j1} - i_{j+1,1}) + L_2(i_{j2} - i_{j+1,2})}{L_C} \tag{3}$$

where $i_{N+1,k} = i_{1k}$ $(k = 1, 2)$. The currents through the nonlinear resistors i_{Rj} are given as

$$i_{Rj} = i_R(v_{j1} + v_{j2}) \tag{4}$$

[1] We have already presented this result in: IEICE Technical Report on Nonlinear Problems (NLP), no. NLP2013-110, pp. 215-218, Oct. 2013, and Proc. of NOLTA, pp. 459-462, Sep. 2013.

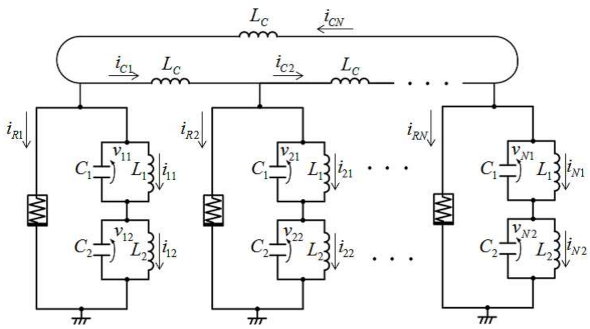

Fig. 1. Ring of coupled simultaneous oscillators with two resonators.

III. Simulation results for five coupled oscillators

In this article, we show several computer simulation results observed from coupled simultaneous oscillators. The results obtained from 5 coupled oscillators are given in this section. A typical example of the observed results is shown in Fig. 2. Figure 2(a) shows attractors observed at 10 resonators. Upper figures correspond to the attractors observed from upper resonators and they have a shape of torus, namely upper resonators exhibit double-mode oscillations. On the other hand, lower figures correspond to lower resonators and they seem to have single frequency. Figure 2(b) shows the phase relationship between horizontally adjacent resonators. Upper figures are almost synchronized at in-phase. While lower figures are also almost synchronized but with some phase differences. Figure 2(c) shows the phase relationship between vertically resonators and that upper and lower resonators are not synchronize. Figure 2(d) shows the time waveforms. From this figure we can observe that the upper resonators generate double-mode oscillations with their envelopes synchronized in 5-phase. Further, oscillations themselves of the upper resonators are synchronized in in-phase even though their amplitudes fluctuate periodically. Moreover, we can see that the lower resonators generate 5-phase synchronizations with almost constant amplitudes. We should also note that the fundamental oscillation frequencies of upper and lower resonators are different, namely they are asynchronous.

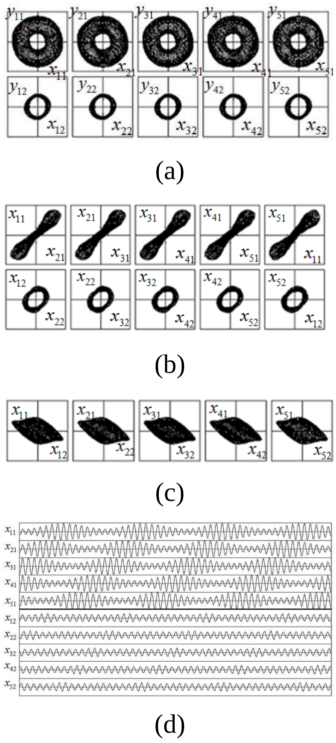

Fig. 2. Typical example of obtained results from 5 coupled oscillators for $\alpha_C = 0.8$, $\alpha_L = 1.0$, $\gamma = 0.3$, $\varepsilon = 0.3$, and $\beta = 3.5$. (a) Attractors observed at 10 resonators. (b) Phase relationship between horizontally adjacent resonators. (c) Phase relationship between vertically adjacent resonators. (c) Time waveforms of the voltages.

IV. Conclusions

In this study, we investigated a ring type of simultaneous oscillators with two LC resonators coupled by inductors. By computer simulations, asynchronous oscillations of double-mode and N-phase were confirmed to be stably generated. We also investigated the effect of changing some important circuit parameters and larger rings of oscillators, but we could not introduce these observed phenomena, because of the limitation of the page space. They will be presented at the conference.

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