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IMPLEMENTATION OF SPECTRAL MUSIC FOR UNIFORM LINEAR ARRAY

Xiang Cao, Jingmin Xin
Xi'an Jiaotong University

Yoshifumi Nishio
The University of Tokushima

1. INTRODUCTION

The multiple signal characterization (MUSIC) algorithm for direction finding has been advanced for many years. To a certain degree, the more search points one use, the more accurate results one will obtain. The main computational burden locate the process of spectral search. In this paper, a polynomial form with real coefficients of spectral music is presented. Our method treats the whole range of interest in uniform linear array. The complexity of spectral music can be reduced by replacing a large number of search points by real roots belonging to $[-1, 1]$.

2. DATA MODEL AND SPECTRAL MUSIC

Assume p far-field narrowband signals $\{s_k\}$ impinging on a Uniform linear array (ULA) with M ($M > p$) sensors. The output vector $\boldsymbol{y}(t)$ of the array at time t can be written as

$$\boldsymbol{y}(t) = \boldsymbol{A}(\boldsymbol{\theta})\boldsymbol{s}(t) + \boldsymbol{n}(t), \tag{1}$$

It is clear that the columns of $\boldsymbol{A}(\boldsymbol{\theta})$ have the maximum projection on the signal subspace $\boldsymbol{U}_s$. With this observation, A natural estimation criterion is to find the local maxima, viz.

$$\hat{\boldsymbol{\omega}} = \arg\max_{\boldsymbol{\omega}} f(\omega) = \arg\max_{\boldsymbol{\omega}} \boldsymbol{a}^H(\omega)\boldsymbol{U}_s\boldsymbol{U}_s^H\boldsymbol{a}(\omega) \tag{2}$$

The total number of complex flops in (2) is $Jp(2M + 1)$, where J ($J \gg M > p$) is the number of spectral search points.

3. REAL POLYNOMIAL FORM OF SPECTRAL MUSIC

By defining a translation

$$z = e^{j(\frac{\omega}{2}+\frac{\pi}{2})}, \tag{3}$$

the real point ω belonging to the domain $[-\pi, \pi]$ can be mapped to the unique complex point z belonging to the unit upper semicircle. Furthermore, let us introduce a conformal transformation

$$u = -j\frac{z-j}{z+j}. \tag{4}$$

The above function is a one-to-one mapping that takes the unit upper semicircle to the interval $[-1, 1]$ of real line. Substituting (3) into (4), we get the relationship between $\omega \in [-\pi, \pi]$ and $u \in [-1, 1]$ as follows:

$$u = \tan\left(\frac{\omega}{4}\right). \tag{5}$$

This is a monotonic function in $[-\pi, \pi]$ and its range is $[-1, 1]$. Thus, once the variable u is estimated, the unique spatial frequency ω can be obtained by $\omega = 4\arctan u$. For this purpose, the array vector response $\boldsymbol{a}(\omega)$ must be connected to u.

Using $z = -j\frac{u-j}{u+j}$ and $e^{j\omega} = -z^2$ (the two equations can be easily obtained from (3) and (4)), we get

$$\boldsymbol{a}(\omega) = (u + j)^{2(1-M)}\boldsymbol{C}\boldsymbol{v}(u). \tag{6}$$

Meanwhile,a function's differential in its local extremum is zero. We can obtain [1]

$$\frac{\partial f(\omega)}{\partial\omega} = 2Re\left\{\boldsymbol{a}^H(\omega)\boldsymbol{W}\boldsymbol{U}_s\boldsymbol{U}_s^H\boldsymbol{a}(\omega)\right\} = 0, \tag{7}$$

where $Re\{\}$ is a real operator and diagonal matrix $\boldsymbol{W} = diag[0, j, \cdots, j(M-1)]$. Applying (6),(7), $\boldsymbol{v}^H(u) = \boldsymbol{v}^T(u)$ and dropping the scalar give

$$\boldsymbol{v}^T(u)Re\left\{\boldsymbol{C}^H\boldsymbol{W}\boldsymbol{U}_s\boldsymbol{U}_s^H\boldsymbol{C}\right\}\boldsymbol{v}(u) = 0. \tag{8}$$

This is an univariate polynomial of degree $4(M-1)$ with real coefficients.

4. CONCLUSIONS

An univariate polynomial with real coefficients is presented for reducing the search points in spectral MUSIC. The real roots belonging to $[-1, 1]$ only need to be solved and the process of solution can avoid all complex arithmetic.

5. REFERENCES

[1] J. Selva, "Computation of spectral and root music through real polynomial rooting," *IEEE Trans. Signal Process.*, vol. 53, pp. 1923–1927, May 2005.